MOTION IN SPACE

1. Suppose the position of an object at time t is given by $\mathbf{r}(t) = \langle 2e^{2t} + 1, e^{2t} - 1, 2e^{2t} - 10 \rangle$.

a) Find the velocity (a vector) and speed (a scalar) of the object.

b) Find the acceleration of the object (a vector).

2. Determine if the trajectory $\mathbf{r}(t) = \langle 3\sin t, 5\cos t, 4\sin t \rangle$ lies on the surface of a sphere and evaluate $\mathbf{r}(t) \cdot \mathbf{v}(t)$. What can you conclude about the angle between $\mathbf{r}(t)$ and $\mathbf{v}(t)$?

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3. In this problem you'll show that your conclusion for the previous problem is part of a larger pattern. An object moving on a sphere of radius c must have a position function $\mathbf{r}(t)$ that satisfies the equation $|\mathbf{r}(t)| = c$.

a) Verify that for any vector $\mathbf{r}$, $\mathbf{r} \cdot \mathbf{r} = |\mathbf{r}|^2$.

b) Use the dot product rule for derivatives to evaluate $\frac{d}{dt}[\mathbf{r}(t) \cdot \mathbf{r}(t)]$ for the object moving on a sphere.

c) Show that $\mathbf{r}(t)$ and $\mathbf{v}(t)$ are orthogonal.

d) Explain what this means in (relatively) normal English.

4. Suppose an object is subject to an acceleration of $\mathbf{a}(t) = \langle 1, t \rangle$ and has initial position $\mathbf{r}(0) = \langle 0, 8 \rangle$ and initial velocity $\mathbf{v}(0) = \langle 2, -1 \rangle$ (with measurements in meters and seconds). Find velocity and position functions for the object.