DERIVATIVES

1. The x-, y-, and z-components of the velocity of a fluid moving in three dimensions are given by the functions u, v, and w. The speed of the fluid at (x, y, z) is then

$$s(x,y,z) = \sqrt{[u(x,y,z)]^2 + [v(x,y,z)]^2 + [w(x,y,z)]^2}.$$

The rate of change of the water speed in the x direction is then

$$\frac{\partial s}{\partial x} = \frac{\partial s}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial s}{\partial v}\frac{\partial v}{\partial x} + \frac{\partial s}{\partial w}\frac{\partial w}{\partial x}.$$

a) Find similar formulas for $\frac{\partial s}{\partial y}$ and $\frac{\partial s}{\partial z}$.

b) Calculate $\frac{\partial s}{\partial x}$, $\frac{\partial s}{\partial y}$, and $\frac{\partial s}{\partial z}$ if $u(x,y,z) = z\cos x$, $v(x,y,z) = z \sin y$, and $w(x,y,z) = 1 - z$. (Look for helpful patterns).

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2. Economists in Absurdistan have determined that Absurdistanis' relative preference for money (x) or leisure time (t) is given by the Cobb-Douglas utility function $\boxed{U(x,t) = x^{0.4}t^{0.6}}$. Higher values are better; if two people have the same amount of money, then the one with more free time will experience higher utility (and be happier). If c is a constant, then the plane curve described by $U(x,t) = c$ is a called a **curve of indifference** (people at different points on the curve are equally content).

a) $\frac{\partial U}{\partial x}$ and $\frac{\partial U}{\partial t}$ are the marginal utility of money and time, respectively. Calculate both.

b) If someone has $x = 1$ and $t = 1$, will giving them more money or more time make them happier faster?

c) Is your answer to part b the same at all points along the curve of indifference $U(x,t) = 1$?

3. Laplace's equation is $\boxed{\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0}$. This equation shows up in studying fluid flow, heat distribution in a conducting medium, and many other places. A function $u(x,y)$ that satisfied Laplace's equation is said to be **harmonic**. Show that the following functions are harmonic.

a) $u(x,y) = e^{-x}\sin y$

b) $u(x,y) = x(x^2 - 3y^2)$