GRADIENTS AND TANGENT PLANES

1. This problem deals with the function $f(x, y) = x^2 - y^2$.

a) Plot $z = x^2 - y^2$ in CalcPlot3D.
b) Add level curves using the small reddish button. Set the step size to 0.25.
c) Calculate $\nabla f(1, \frac{1}{2})$.
d) Find a parametric equation for the line in the plane $z = \frac{3}{4}$ passing through the point $\left(1, \frac{1}{2}, \frac{3}{4}\right)$ and having direction vector $\nabla f(1, \frac{1}{2})$.
e) Add the line to your contour plot and observe that it is orthogonal to the level curve at the point $\left(1, \frac{1}{2}, \frac{3}{4}\right)$.
f) This means the tangent to the curve can be described by $\left(\langle x, y\rangle - \langle 1, \frac{1}{2}\rangle\right) \cdot \nabla f(1, \frac{1}{2}) = 0$. Solve this for y and then find a parametric description of the tangent vector to the level curve at the point $\left(1, \frac{1}{2}, \frac{3}{4}\right)$. Add it to your plot. Admire.

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2. The surface in problem 1 can be thought of as the level surface $w = 0$ where $w = x^2 - y^2 - z$.

a) Calculate $\nabla w(1, \frac{1}{2}, \frac{3}{4})$ and add this vector to your graph at initial point $(1, \frac{1}{2}, \frac{3}{4})$. This vector should be orthogonal to the surface (in particular, to the tangent line you added to the plot in part f; 3-d glasses might help).

b) The tangent plane to the surface at the point $(1, \frac{1}{2}, \frac{3}{4})$ is described by

$$\left(\langle x, y, z\rangle - \left\langle 1, \frac{1}{2}, \frac{3}{4}\right\rangle\right) \cdot \nabla w\left(1, \frac{1}{2}, \frac{3}{4}\right) = 0.$$

Do the algebra necessary to add this plane to your plot and observe its tangency (and its normality to the vector you added in the last step).