INTEGRALS APPLIED

Definition. The z coordinate of the center of mass of a solid with density $f(x,y,z)$ occupying region D is

$$\overline{z} = \frac{1}{m} \iiint_D z f(x,y,z)\; dV$$

where $m = \iiint_D f(x,y,z)\; dV$ is the mass of the solid.

1. Find the center of mass of the constant-density solid cone bounded by $z = 4 - \sqrt{x^2+y^2}$ and $z = 0$.

2. The hemisphere D occupies the region inside $x^2+y^2+z^2 = 1$ with $z \geq 0$. Find the center of mass of D if its density is $f(x,y,z) = 2 - \sqrt{x^2+y^2+z^2}$.

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3. Find the volume of the solid formed by the intersection of the cylinders $x^2 + z^2 = 1$ and $y^2 + z^2 = 1$.

4. Find the volume of the solid formed by the intersection of the cylinders $x^2 + y^2 = 1$, $x^2 + z^2 = 1$, and $y^2 + z^2 = 1$.