Math 259

FINAL EXAM

INSTRUCTIONS: Solve 13 of the 14 problems; choose one problem to skip and put a big \mathbf{X} in the corresponding box below. If you do not indicate which problem you are choosing to skip, I will assume that I should skip problem 14, which may hurt your grade. Show your work: even correct answers may receive little or no credit if a method of solution is not shown. You do not need to simplify your solutions. Graphing calculators, notes, cell phones, and other materials are not permitted.

Name:										Total:			
1	2	3	4	5	6	7	8	9	10	11	12	13	14

1. Find an equation for the line the tangent to the curve $\mathbf{r}(t) = \langle \sin(\pi t), t^2, 1-t \rangle$ at the point (0, 1, 0).

2. Use the two-path test to explain why the limit does not exist: $\lim_{(x,y)\to(0,0)} \frac{4xy}{3x^2+y^2}$

3. Find a unit vector perpendicular to both of the the lines $\mathbf{r}_1(t) = \langle 2t, 3, t+1 \rangle$ and $\mathbf{r}_2(t) = \langle 1-t, 3t, t \rangle$.

4. Determine if the planes -x + 2y + 3z = 6 and 4x - y + 2z = 4 are parallel, perpendicular, or neither.

5. Match the contour plot to the function. Explain your reasoning for possible partial credit.



6. Find $\frac{\partial w}{\partial t}$ if w = 2xy + 3yz, $x = e^{st}$, $y = \sin t$, and $z = s^2 t^3$.

7. Find all the critical points of the function $f(x, y) = x^3y + 12x^2 - 8y$ and use the second derivative test to determine if each is a local minimum, local maximum, saddle, or indeterminate.

8. Find the maximum rate of change of the function f(x, y, z) = 1 + 3xyz at the point (1, -1, 2) and the direction in which it occurs (the direction does not need to be a unit vector).

9. A contour plot for z = f(x, y) is shown together with a constraint curve g(x, y) = c. Use the plot to estimate the maximum and minumum values of f(x, y) given the constraint g(x, y) = c.



10. Evaluate the integral (change the order of integration if necessary): $\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3 + 1} \, dx \, dy$

11. Evaluate the integral $\iiint_E z \, dV$ where *E* is the region between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ in the first octant (*x*, *y*, and *z* are all positive).

12. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ for $\mathbf{F} = \langle e^{2y}, 1 + 2xe^{2y} \rangle$ and $C : \mathbf{r}(t) = \langle te^t, 1+t \rangle$ for $0 \le t \le 1$. Hint: is the vector field conservative?

13. Let $\mathbf{F} = \langle xy, 4x^2, yz \rangle$ and let S be the surface $z = xe^y$ over the square $0 \le x \le 1$ and $0 \le y \le 1$. Calculate the flux of \mathbf{F} across S. That is, evaluate the integral $\iint_S \mathbf{F} \cdot \mathbf{n} \, dS$. 14. Let $\mathbf{F} = \langle y^2, 3xy \rangle$ and let C be the boundary of the semi-circle $x^2 + y^2 = 4$ with $y \ge 0$ (oriented counterclockwise). Use Green's theorem to evaluate the integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$.

