VECTORS

1. Let P be the point $(-1, 2)$, Q be the point $(0, -1)$, and R be the point $(2, 3)$.

a) Sketch the vectors $\overrightarrow{PQ}$, $\overrightarrow{PR}$, and $\overrightarrow{QR}$.

b) Calculate the components of the 3 vectors, e.g. $\overrightarrow{PQ} = \langle 1, -3 \rangle$.

c) Verify that the triangle rule for addition (adding with pictures) agrees with component addition (adding components) and $\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$.

2. The distance between points a and b in $\mathbb{R}$ is $\sqrt{(b-a)^2}$. The distance between points (a_1, a_2) and (b_1, b_2) in $\mathbb{R}^2$ is $\sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2}$.

a) Find a formula for the distance between points (a_1, a_2, a_3) and (b_1, b_2, b_3) in $\mathbb{R}^3$.

b) Find a formula for the distance between points (a_1, a_2, a_3, a_4) and (b_1, b_2, b_3, b_4) in $\mathbb{R}^4$.

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Definition. A vector $\mathbf{v}$ is a **unit vector** if it has magnitude 1 (in symbols: $|\mathbf{v}| = 1$).

3. Adding an extra component to a 2-component vector gives us a vector in 3 dimensions.

a) Find the vector from $P(1, 2, 3)$ to $Q(-3, 5, -2)$.

b) Calculate the magnitude of the vector $\overrightarrow{PQ}$.

c) Find a unit vector parallel to $\overrightarrow{PQ}$.

d) How many different unit vectors are parallel to $\overrightarrow{PQ}$?

4. Let $\mathbf{u} = \langle 1, 2, 3\rangle$ and $\mathbf{v} = \langle -1, -2, 3\rangle$. Calculate the following.

a) $\mathbf{u} + \mathbf{v}$.

b) $\mathbf{u} - \mathbf{v}$.

c) $2\mathbf{u} + \mathbf{v}$.

5. A drone is flying horizontally due north at a speed of 5 mi/hr when it encounters a horizontal crosswind blowing northwest at 8 mi/hr and an updraft blowing up at 1 mi/hr.

a) Find the velocity vector of the drone immediately after it encounters the winds.

b) Find the speed of the drone.

6. Describe (or draw) the set of points x in $\mathbb{R}$ that satisfy the equation $(x-2)^2 = 9$. Hint: interpret the equation as a statement about the squared distance from 2 to x (see problem 2).

7. Describe (or draw) the set of points (x, y) in $\mathbb{R}^2$ that satisfy the equation $(x-2)^2 + (y-1)^2 = 9$. Hint: interpret the equation as a statement about the squared distance from $(2, 1)$ to x (see problem 2).

8. Describe (or draw) the set of points (x, y, z) in $\mathbb{R}^3$ that satisfy the equation $(x-2)^2+(y-1)^2+z^2=9$. Hint: keep on interpreting the equation as a statement about distance.

9. Describe (or draw) the set of points (x, y, z) in $\mathbb{R}^3$ that satisfy the equation $(x-2)^2+(y-1)^2+z^2\leq 9$.

10. Describe the set of points (x, y, z, w) in $\mathbb{R}^4$ that satisfy the equation $(x-2)^2+(y-1)^2+z^2+(w+1)^2=9$. Hint: keep on interpreting the equation as a statement about distance.