VECTOR PRODUCTS

Definition. Let $\mathbf{u}$ and $\mathbf{v}$ be vectors and let θ be the angle between $\mathbf{u}$ and $\mathbf{v}$ (with $0 \leq \theta \leq \pi$).

(1) The dot product of $\mathbf{u}$ and $\mathbf{v}$ is $\boxed{\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}|\,|\mathbf{v}| \cos\theta}$

(2) The orthogonal projection of $\mathbf{u}$ onto $\mathbf{v}$ is $\boxed{\mathbf{proj}_{\mathbf{v}}\mathbf{u} = \left(\dfrac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v}}$

(3) The scalar projection of $\mathbf{u}$ onto $\mathbf{v}$ is $\boxed{\mathrm{scal}_{\mathbf{v}}\mathbf{u} = \dfrac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|}}$

Theorem. *If* $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ *and* $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$, *then* $\boxed{\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3}$

1. Let $\mathbf{u} = \langle 1, 2, 3 \rangle$ and $\mathbf{v} = \langle 0, -2, 1 \rangle$.

a) Find $\mathrm{scal}_{\mathbf{v}}\mathbf{u}$.

b) Determine if $\mathrm{scal}_{\mathbf{u}}\mathbf{v}$ is positive or negative without doing any additional calculations.

c) Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

2. You probably already know that work is force times distance. However, this only applies when the force and the displacement are in the same direction. The more general version is $\boxed{W = \mathbf{F} \cdot \mathbf{d}}$ force and displacement are vectors. Use this to calculate the work done by a ski lift that moves 200 kg of skiers along the vector $\langle 80, 350, 300 \rangle$ (measured in meters) while subject to the force of gravity and wind exerting a force of $\langle 15, -120, -60 \rangle$ Newtons.

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Definition. If $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$, then the **cross product** of $\mathbf{u}$ and $\mathbf{v}$ is

$$\boxed{\mathbf{u} \times \mathbf{v} = \langle u_2 v_3 - u_3 v_2,\ u_3 v_1 - u_1 v_3,\ u_1 v_2 - u_2 v_1 \rangle}$$

(Note that the cross product is only defined for 3-dimensional vectors.)

3. The coordinate vectors of $\mathbb{R}^3$ are $\mathbf{i} = \langle 1, 0, 0 \rangle$, $\mathbf{j} = \langle 0, 1, 0 \rangle$, and $\mathbf{k} = \langle 0, 0, 1 \rangle$.

a) Find $\mathbf{i} \times \mathbf{j}$

b) Find $\mathbf{j} \times \mathbf{i}$

c) Find $\mathbf{j} \times \mathbf{k}$

d) Find two vectors whose cross product is $\mathbf{j}$