MOTION IN SPACE

1. Suppose the position of an object at time t is given by $\mathbf{r}(t) = \langle 2e^{2t} + 1, e^{2t} - 1, 2e^{2t} - 10 \rangle$.

a) Find the velocity (a vector) and speed (a scalar) of the object when (both are functions of t).

b) Find the acceleration of the object (a vector, also a function of t).

2. A trajectory $\mathbf{r}(t)$ lies on the surface of a sphere of radius c centered at the origin if $|\mathbf{r}(t)| = c$. Determine if the trajectory $\mathbf{r}(t) = \langle 3\sin t, 5\cos t, 4\sin t \rangle$ lies on the surface of a sphere and evaluate $\mathbf{r}(t) \cdot \mathbf{v}(t)$. What can you conclude about the angle between $\mathbf{r}(t)$ and $\mathbf{v}(t)$?

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3. In this problem you'll show that your conclusion about the angle between $\mathbf{r}(t)$ and $\mathbf{v}(t)$ in the previous problem is part of a larger pattern.

a) Use the dot product rule for derivatives to evaluate $\frac{d}{dt}[\mathbf{r}(t) \cdot \mathbf{r}(t)]$ and express the answer in terms of $\mathbf{r}(t) \cdot \mathbf{v}(t)$.

b) Verify that for any vector $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$, $\mathbf{u} \cdot \mathbf{u} = |\mathbf{u}|^2$

c) Suppose that the trajectory $\mathbf{r}(t)$ lies on the surface of a sphere of radius c centered at the origin. What does this mean about $\mathbf{r}(t) \cdot \mathbf{r}(t)$? What does this tell you about $\frac{d}{dt}[\mathbf{r}(t) \cdot \mathbf{r}(t)]$?

d) Show that $\mathbf{r}(t)$ and $\mathbf{v}(t)$ are orthogonal.

e) Explain what this means in (relatively) normal English.