PLANES AND SURFACES

We have seen that an equation for the plane containing the point $\mathbf{r}_0 = \langle x_0, y_0, z_0 \rangle$ with normal vector $\mathbf{n} = \langle a, b, c \rangle$ is $\boxed{\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0}$ or, equivalently, $\boxed{a(x - x_0) + b(y - y_0) + c(z - z_0) = 0}$.

1. Consider the plane containing the points $P(1, 2, 3)$, $Q(-1, 0, 3)$, and $R(0, 3, 2)$.

a) Find an equation for the plane.

b) Find an equation for a parallel plane containing the origin. Hint: what connection is there between the normal vector for this plane and the normal vector for the plane in part a?

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If two planes in $\mathbb{R}^3$ are not parallel, then they intersect in a line. That line lies in both planes, hence its direction vector must be orthogonal both of the normal vectors for the planes.

2. Find the line of intersection of the planes $x - y - 2z = 0$ and $x + 2y - z = 1$.

A. Discussion questions

3. Plot the following surfaces in CalcPlot3D (using the **Implicit Surface** item in the **Add to graph** menu. What can you say about the intersection of the surface with the given coordinate plane?

a) $x^2 + y^2 = 1$ (xy-plane)
b) $x^2 + z = 1$ (xz-plane)
c) $y^2 - z^2 = 1$ (yz-plane)

The surfaces in the previous problem are all examples of **cylinders** (see the definition on P. 862). Generally, you can figure out what a cylinder looks like by plotting its intersection with a single coordinate plane. More complicated surfaces can be understood by plotting their intersections with all 3 coordinate planes.

4. Plot the intersection of the given surface with each coordinate plane and guess at the shape of the surface (try to sketch the surface in your notebook). Then plot the surface itself.

a) $x^2 - y^2 - z = 0$
b) $x^2 + y^2 - z = 0$
c) $x^2 + y^2 - z^2 = 0$
d) $x^2 + y^2 - z^2 = 1$
e) $x^2 + y^2 - z^2 = -1$

Another good way to understand a surface of the form $z = f(x, y)$ is through intersections of the surface with the planes $z = c$ for different constants c. The resultant curves are called **level curves**.

5. Plot the level curves of $z = \dfrac{1}{x^2 + y^2}$ at heights $z = 1, 3, 5, 7, 9$.