The mass of a lamina with density $\rho(x,y)$ occupying region $D$ in the $xy$-plane is

$$m = \int\int_D \rho(x,y) \, dA.$$ 

The moments about the $x$-axis and $y$-axis of the same lamina are (respectively)

$$M_x = \int\int_D y\rho(x,y) \, dA \quad \text{and} \quad M_y = \int\int_D x\rho(x,y) \, dA.$$ 

The center of mass of the lamina is $(\overline{x}, \overline{y})$ where $\overline{x} = \frac{M_y}{m}$ and $\overline{y} = \frac{M_x}{m}$.

1. Find the center of mass of a lamina with density $\rho(x,y) = y$ occupying the triangular region bounded by $x = 0$, $y = x$, and $y = 2 - x$. 

The moment of inertia (also called the second moment) of a particle of mass $m$ at distance $r$ from the axis of rotation is $mr^2$. For a lamina with density $\rho(x,y)$ occupying region $D$ in the $xy$-plane, the moments of inertia about the $x$-axis and $y$-axis are (respectively)

$$I_x = \int\int_D y^2\rho(x,y) \, dA \quad \text{and} \quad I_y = \int\int_D x^2\rho(x,y) \, dA.$$ 

The moment of inertia about the origin is

$$I_0 = \int\int_D (x^2 + y^2)\rho(x,y) \, dA.$$
2. Find the moments of inertia \((I_x, I_y, \text{ and } I_0)\) of a lamina with density \(\rho(x, y) = x\) and occupying \(D = \{(x, y) \mid x \geq 0, y \geq 0, x^2 + y^2 \leq 1\}\).

3. Find the center of mass of the solid of uniform density occupying the region below the surface \(z = 1 - x^2 - y^2\), above the \(xy\)-plane, and inside the circle \(x^2 + y^2 = x\).