1. A solid occupies the region above the plane $z = 1$, below the plane $z = y$, and inside the cylinder $x^2 + y^2 = 4$.

   a) Express the volume of the solid as an integral in Cartesian coordinates.

      Solution. $V = \int_{-\sqrt{3}}^{\sqrt{3}} \int_{1}^{\sqrt{4-x^2}} \int_{1}^{\sqrt{y}} dz
dy
dx$ (other orders of integration are possible).

   b) Express the volume of the solid as a triple integral in cylindrical coordinates. Hint: $\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$.

      Solution. $V = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_{0}^{2\pi} \int_{1}^{\sqrt{4-r^2}} r
dz
dr
d\theta$ (other orders of integration are possible).

   c) Calculate the volume of the solid. It may be helpful to recall that $\int \csc^2 u
du = -\cot u + C$.

      Solution. Evaluating either the Cartesian or cylindrical integral gives $V = 3\sqrt{3} - \frac{4}{3}\pi$.

2. A solid (the “sno-cone”) occupies the region above $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 4$.

   a) Express the volume of the solid as an integral in cylindrical coordinates.

      Solution. $V = \int_{0}^{2\pi} \int_{0}^{\sqrt{2}} \int_{\sqrt{4-r^2}}^{2} r
dz
dr
d\theta$ (other orders of integration are possible).

   b) Express the volume of the solid as an integral in spherical coordinates.

      Solution. $V = \int_{0}^{\frac{\pi}{4}} \int_{0}^{2\pi} \int_{0}^{2} \rho^2 \sin \phi
d\rho
d\phi
d\phi$ (other orders of integration are possible).

   c) Calculate the volume of the solid.

      Solution. Evaluating either the cylindrical or spherical integral gives $V = \frac{16}{3}\pi \left( 1 - \frac{1}{\sqrt{2}} \right)$.