GRADIENTS AND TANGENT PLANES

1. This problem deals with the function $f(x, y) = x^2 - y^2$.

a) Plot the surface $z = x^2 - y^2$ in CalcPlot3D.

b) Add level curves using the small red level curve button. **Important:** Set the step size to 0.25. Locate the level curve $z = 0.75$ and observe that because $f(1, 1/2) = 3/4$, the point $(1, 1/2)$ is on this level curve.

c) Calculate $\nabla f(1, 1/2)$.

d) Find a parametric equation for the line in the plane $z = 3/4$ passing through the point $\left(1, \frac{1}{2}, \frac{3}{4}\right)$ and having direction vector $\nabla f(1, 1/2)$.

e) Add the line from the last part to your contour plot (select **Space Curve: r(t)**). At what angle does it meet the level curve $z = 0.75$?

f) It follows that the line tangent to the level curve $z = 0.75$ can be described by

$$\left(\langle x, y \rangle - \left\langle 1, \frac{1}{2} \right\rangle \right) \cdot \nabla f \left(1, \frac{1}{2} \right) = 0$$

Solve this equation for y and then find a parametric description of the line (in the plane $z = 3/4$). Add this parametric curve to your plot and admire.

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2. The surface in problem 1 can be thought of as a level surface of $g(x, y, z) = x^2 - y^2 - z$.

a) Which level surface is the correct level surface? That is, what value of $w = g(x, y, z)$ gives an equation that is the same as $z = x^2 - y^2$?

b) Calculate $\nabla g(1, \frac{1}{2}, \frac{3}{4})$ and add this vector to your graph at initial point $(1, \frac{1}{2}, \frac{3}{4})$. This vector should be orthogonal to the surface (in particular, to the tangent line you added to the plot in part 1f).

c) The **tangent plane** to the surface at the point $(1, \frac{1}{2}, \frac{3}{4})$ is described by

$$\left(\langle x, y, z\rangle - \left\langle 1, \frac{1}{2}, \frac{3}{4}\right\rangle\right) \cdot \nabla g\left(1, \frac{1}{2}, \frac{3}{4}\right) = 0.$$

Do the algebra necessary to add this plane to your plot and observe its tangency (and its normality to the vector you added in the last step).