MAXIMUM AND MINIMUM VALUES

1. Find the local maximum and minimum values of $f(x, y) = x^4 + y^4 - 4xy + 1$.

2. The goal of this problem is to find the maximum and minimum values of $f(x, y) = x^2 - y^2$ on the circular region $x^2 + y^2 \leq 1$.

a) Find any critical points of $f(x, y)$ that are inside the region (you do not need to use the second derivative test to identify their type).

b) Find the critical points on the boundary of the region by substituting $y^2 = 1 - x^2$ into $f(x, y) = x^2 - y^2$ and then finding the critical points of the resultant one-variable function.

c) Evaluate $f(x, y)$ at all of your critical points; the largest value is the maximum and the smallest value is the minimum.

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3. The goal of this problem is to find the minimum distance from the plane $x + 2y + z = 4$ to the point $(1, 0, -2)$.

a) Instead of minimizing distance, we'll minimize squared distance. Find a function for the squared distance from (x, y, z) to $(1, 0, -2)$.

b) Our points (x, y, z) must lie in the plane $x+2y+z = 4$. Incorporate this constraint by substituting $z = 4 - x - 2y$ into your function from part a.

c) Find the global minimum of the resultant function (which should have 2 variables).

The method of the previous problem is sometimes difficult to implement, but there's an alternative:

Method (Lagrange multipliers)**.** To find maximum and minimum values of $f(x, y, z)$ subject to the constraint $g(x, y, z) = c$:

(1) Find all possible solutions to $\nabla f(x, y, z) = \lambda \nabla g(x, y, z)$ and $g(x, y, z) = c$. (This gives you 4 equations in 4 variables; ingenuity might be required).

(2) Evaluate $f(x, y, z)$ at all the points. The largest value you get is the max and the smallest value is the min.

Challenge. A rectangular box without a top is to be made from 12 m^2 of material. Our goal is to find the largest possible volume of the box.

a) Find the function to be maximized.

b) Find the constraint function.

c) Use the method of Lagrange multipliers.

Challenge. Find the minimum and maximum distances from the sphere $x^2 + y^2 + z^2 = 4$ to the point $(3, 1, -1)$. Hint: this can be achieved either using Lagrange multipliers or geometric ingenuity.