

TRIPLE INTEGRALS

1. Express the integral $\int_0^2 \int_0^{2-z} \int_0^{\frac{y}{2}} f(x, y, z) \, dx \, dy \, dz$ as an integral in the order $dz \, dy \, dx$. Hint: find the volume of integration by first finding its projection into the yz -plane (as described by the outer two integrals).

2. Let D be the solid above the xy -plane, below the plane $z = 2 + y$, and inside the cylinder $x^2 + y^2 = 4$.

a) Express the volume of D as a triple integral with dz first (the innermost integral).

b) Evaluate just the dz integral.

c) You now have a double integral. Convert this to a polar integral.

d) Evaluate the polar integral.

3. Find the mass of the solid with density $f(x, y, z) = 1 + z$ occupying the region above the xy -plane and below the surface $z = 1 - x^2 - y^2$.

4. A solid occupies the region above $z = \sqrt{x^2 + y^2}$ and below $x^2 + y^2 + z^2 = 4$ (which makes a sort of sno-cone). Find a triple integral for the volume of the solid.