INTEGRALS APPLIED

Definition. The center of mass of a solid with density $f(x, y, z)$ occupying region D is $(\overline{x}, \overline{y}, \overline{z})$ where

$$\overline{x} = \frac{1}{m} \iiint_D x f(x, y, z)\, dV$$

$$\overline{y} = \frac{1}{m} \iiint_D y f(x, y, z)\, dV$$

$$\overline{z} = \frac{1}{m} \iiint_D z f(x, y, z)\, dV$$

$$\text{and } m = \iiint_D f(x, y, z)\, dV \text{ is the mass of the solid.}$$

1. Find the center of mass of a constant-density solid half-cone cone:

$$D = \{(x, y, z) : 0 \leq z \leq 4 - \sqrt{x^2 + y^2} \text{ and } y \geq 0\}$$

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2. The hemisphere D occupies the region inside $x^2 + y^2 + z^2 = 1$ with $z \geq 0$. Find the center of mass of D if its density is $f(x, y, z) = 2 - \sqrt{x^2 + y^2 + z^2}$. Hint: take advantage of the symmetry of the solid and its density.

Challenge. Find the volume of the solid formed by the intersection of the cylinders $x^2 + z^2 = 1$ and $y^2 + z^2 = 1$. Hint: What does the solid look like from above? Take advantage of symmetry.

Challenge. Find the volume of the solid formed by the intersection of the cylinders $x^2+y^2 = 1$, $x^2+z^2 = 1$, and $y^2 + z^2 = 1$.