Math 260 **Exam 1** **February 7, 2018**

Instructions: Solve $N-1$ of the following N problems and write your solutions on the provided paper, clearly labeling each solution. All solutions should include a clear method or argument and should use English words and sentences when appropriate. Clear and comprehensible solutions will generally earn more points than those that are hard to understand and a correct solution without supporting work may receive little or no credit. Indicate which problem you are skipping by placing an **X** in the corresponding box below. Leave the rest blank (I'll use them to record your scores).

Calculators, phones, and all other devices are forbidden. Answers may be left unsimplified.

Name:									
1	2	3	4	5	6	7	...	N	Total

Thoerem. *If the characteristic polynomial of $ay'' + by' + cy' = 0$ has...*

a) ...distinct real roots r_1 and r_2, then a general solution is $y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$

b) ...a single (repeated) real root r, then a general solution is $y = e^{rx}(c_1 + c_2 x)$

c) ...complex conjugate roots $\lambda \pm i\omega$ (where $\omega > 0$), then a general solution is $y = e^{\lambda x}(c_1 \cos\omega x + c_2 \sin\omega x)$

Thoerem. *If y_p is any particular solution to the differential equation $y'' + p(x)y' + q(x)y = f(x)$ and $\{y_1, y_2\}$ is a fundamental set of solutions to the complementary equation, then a general solution for differential equation is*

$$y = y_p + c_1 y_1 + c_2 y_2$$

Thoerem (Superposition). *If y_{p_1} is a particular solution of $y'' + p(x)y' + q(x)y = f_1(x)$ and y_{p_2} is a particular solution of $y'' + p(x)y' + q(x)y = f_2(x)$, then a particular solution of*

$$y'' + p(x)y' + q(x)y = f_1(x) + f_2(x)$$

is

$$y_p = y_{p_1} + y_{p_2}$$

Method. To find a particular solution for $ay'' + by' + cy = e^{\alpha x}G(x)$ (where G is a polynomial), solve for the coefficents of $Q(x)$ in the following (where Q is a polynomial with the same degree as G):

a) $y_p = e^{\alpha x}Q(x)$ if $e^{\alpha x}$ is not a solution to the complementary equation;

b) $y_p = xe^{\alpha x}Q(x)$ if $xe^{\alpha x}$ is not a solution to the complementary equation, but $e^{\alpha x}$ is a solution to the complementary equation;

c) $y_p = x^2 e^{\alpha x}Q(x)$ if $xe^{\alpha x}$ and $e^{\alpha x}$ are both solutions to the complementary equation.

Method. To find a particular solution for $ay'' + by' + cy = P(x)\cos\omega x + Q(x)\sin\omega x$ (where P and Q are polynomials), solve for the coefficents of $A(x)$ and $B(x)$ in the following (where A and B are polynomials with degree equal to the larger of the degrees of P and Q):

a) $y_p = A(x)\cos\omega x + B(x)\sin\omega x$ if $\cos\omega x$ and $\sin\omega x$ are not solutions to the complementary equation;

b) $y_p = x\left[A(x)\cos\omega x + B(x)\sin\omega x\right]$ if $\cos\omega x$ and $\sin\omega x$ are solutions to the complementary equation;