Math 260 **Exam 3** **April 16, 2018**

Instructions: Solve 7 of the following 8 problems and write your solutions on the provided paper, clearly labeling each solution (do not write your solutions on this sheet). All solutions should include a clear method or argument and should use English words and sentences when appropriate. Clear and comprehensible solutions will generally earn more points than those that are hard to understand; a correct solution without supporting work may receive little or no credit. Indicate which problem you are skipping by placing an **X** in the corresponding box below. Leave the rest blank (I'll use them to record your scores).

Calculators, phones, and all other devices are forbidden. Answers may be left unsimplified.

Name:								
1	2	3	4	5	6	7	8	Total

Definition. The **Laplace transform** of f is $\boxed{L(f) = F(s) = \int_0^\infty f(t)e^{-st}dt}$

Thoerem (First Shifting Theorem)**.** *If* $L(f) = F(s)$, *then* $\boxed{L\left(e^{at}f(t)\right) = F(s-a)}$

Thoerem. $\boxed{L(f') = sL(f) - f(0)}$ *and* $\boxed{L(f'') = s^2L(f) - sf(0) - f'(0)}$

Thoerem. $f(t) = \begin{cases} f_0(t), & 0 \le t < t_0 \\ f_1(t), & t \ge t_0 \end{cases} = f_0(t) + u(t-t_0)\left[f_1(t) - f_0(t)\right]$

Thoerem (Second Shifting Theorem)**.**

1. $\boxed{L\left(u(t-t_0)g(t)\right) = e^{-t_0 s}L\left(g(t+t_0)\right)}$
2. $\boxed{L\left(u(t-t_0)g(t-t_0)\right) = e^{-t_0 s}L\left(g\right)}$

Definition. The convolution of functions f and g is the function $f * g$ defined by $\boxed{(f*g)(t) = \int_0^t f(\tau)g(t-\tau)d\tau}$

Thoerem (Convolution Theorem)**.** *If* $L(f) = F$ *and* $L(g) = G$, *then* $\boxed{L(f*g) = FG}$

Definition. The solution to the initial value problem $ay'' + by' + cy = \delta(t-t_0),\ y(0) = 0,\ y'(0) = 0$ is $\boxed{y = u(t-t_0)w(t-t_0)}$ where $w(t) = L^{-1}\left(\dfrac{1}{as^2 + bs + c}\right)$.

Thoerem (Superposition)**.** *If* y_1 *is the solution of the IVP*

$$ay'' + by' + cy = f_1(t),\ y(0) = k_1,\ y'(0) = k_2$$

and y_2 *is the solution of the IVP*

$$ay'' + by' + cy = f_2(t),\ y(0) = l_1,\ y'(0) = l_2,$$

then $y_1 + y_2$ *is the solution to the IVP*

$$ay'' + by' + cy = f_1(t) + f_2(t),\ y(0) = k_1 + l_1,\ y'(0) = k_2 + l_2.$$

Laplace transforms

$f(t)$	1	t^n	e^{at}	$t^n e^{at}$	$\sin \omega t$	$\cos \omega t$	$\sinh bt$	$\cosh bt$	$\delta(t-t_0)$
$F(S)$	$\frac{1}{s}$	$\frac{n!}{s^{n+1}}$	$\frac{1}{s-a}$	$\frac{n!}{(s-a)^{n+1}}$	$\frac{\omega}{s^2+\omega^2}$	$\frac{s}{s^2+\omega^2}$	$\frac{b}{s^2-b^2}$	$\frac{s}{s^2-b^2}$	$e^{-t_0 s}$