16. Use the ideas developed in Exercise 15 to find infinitely many solutions of the initial value problem

$$y' = y^{2/5}, \quad y(0) = 1$$

on $(-\infty, \infty)$.

17. Consider the initial value problem

$$y' = 3x(y-1)^{1/3}, \quad y(x_0) = y_0. \tag{A}$$

(a) For what points (x_0, y_0) does Theorem 2.3.1 imply that (A) has a solution?

(b) For what points (x_0, y_0) does Theorem 2.3.1 imply that (A) has a unique solution on some open interval that contains x_0?

18. Find nine solutions of the initial value problem

$$y' = 3x(y-1)^{1/3}, \quad y(0) = 1$$

that are all defined on $(-\infty, \infty)$ and differ from each other for values of x in every open interval that contains $x_0 = 0$.

19. From Theorem 2.3.1, the initial value problem

$$y' = 3x(y-1)^{1/3}, \quad y(0) = 9$$

has a unique solution on an open interval that contains $x_0 = 0$. Find the solution and determine the largest open interval on which it's unique.

20. **(a)** From Theorem 2.3.1, the initial value problem

$$y' = 3x(y-1)^{1/3}, \quad y(3) = -7 \tag{A}$$

has a unique solution on some open interval that contains $x_0 = 3$. Determine the largest such open interval, and find the solution on this interval.

(b) Find infinitely many solutions of (A), all defined on $(-\infty, \infty)$.

21. Prove:

(a) If

$$f(x, y_0) = 0, \quad a < x < b, \tag{A}$$

and x_0 is in (a, b), then $y \equiv y_0$ is a solution of

$$y' = f(x, y), \quad y(x_0) = y_0$$

on (a, b).

(b) If f and f_y are continuous on an open rectangle that contains (x_0, y_0) and (A) holds, no solution of $y' = f(x, y)$ other than $y \equiv y_0$ can equal y_0 at any point in (a, b).

2.4 TRANSFORMATION OF NONLINEAR EQUATIONS INTO SEPARABLE EQUATIONS

In Section 2.1 we found that the solutions of a linear nonhomogeneous equation

$$y' + p(x)y = f(x)$$

are of the form $y = uy_1$, where y_1 is a nontrivial solution of the complementary equation

$$y' + p(x)y = 0 \tag{2.4.1}$$

and u is a solution of

$$u'y_1(x) = f(x).$$

Note that this last equation is separable, since it can be rewritten as

$$u' = \frac{f(x)}{y_1(x)}.$$

In this section we'll consider nonlinear differential equations that are not separable to begin with, but can be solved in a similar fashion by writing their solutions in the form $y = uy_1$, where y_1 is a suitably chosen known function and u satisfies a separable equation. We'llsay in this case that we *transformed* the given equation into a separable equation.

Bernoulli Equations

A *Bernoulli equation* is an equation of the form

$$y' + p(x)y = f(x)y^r, \tag{2.4.2}$$

where r can be any real number other than 0 or 1. (Note that (2.4.2) is linear if and only if $r = 0$ or $r = 1$.) We can transform (2.4.2) into a separable equation by variation of parameters: if y_1 is a nontrivial solution of (2.4.1), substituting $y = uy_1$ into (2.4.2) yields

$$u'y_1 + u(y_1' + p(x)y_1) = f(x)(uy_1)^r,$$

which is equivalent to the separable equation

$$u'y_1(x) = f(x)\,(y_1(x))^r\,u^r \quad \text{or} \quad \frac{u'}{u^r} = f(x)\,(y_1(x))^{r-1},$$

since $y_1' + p(x)y_1 = 0$.

Example 2.4.1 Solve the Bernoulli equation

$$y' - y = xy^2. \tag{2.4.3}$$

Solution Since $y_1 = e^x$ is a solution of $y' - y = 0$, we look for solutions of (2.4.3) in the form $y = ue^x$, where

$$u'e^x = xu^2e^{2x} \quad \text{or, equivalently,} \quad u' = xu^2e^x.$$

Separating variables yields

$$\frac{u'}{u^2} = xe^x,$$

and integrating yields

$$-\frac{1}{u} = (x-1)e^x + c.$$

Hence,

$$u = -\frac{1}{(x-1)e^x + c}$$

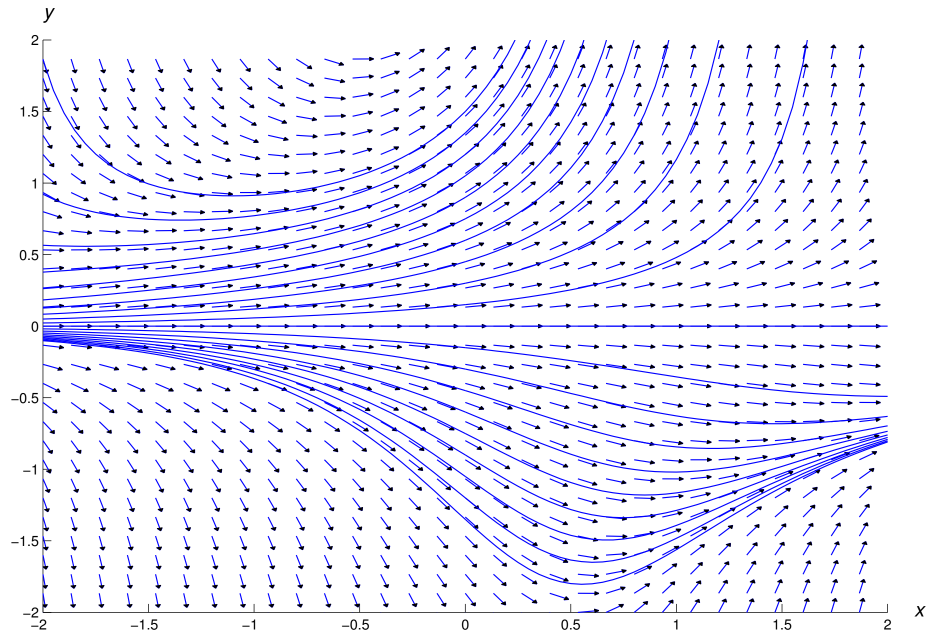

Figure 2.4.1 A direction field and integral curves for $y' - y = xy^2$

and

$$y = -\frac{1}{x - 1 + ce^{-x}}.$$

Figure 2.4.1 shows direction field and some integral curves of (2.4.3).

Other Nonlinear Equations That Can be Transformed Into Separable Equations

We've seen that the nonlinear Bernoulli equation can be transformed into a separable equation by the substitution $y = uy_1$ if y_1 is suitably chosen. Now let's discover a sufficient condition for a nonlinear first order differential equation

$$y' = f(x, y) \tag{2.4.4}$$

to be transformable into a separable equation in the same way. Substituting $y = uy_1$ into (2.4.4) yields

$$u'y_1(x) + uy_1'(x) = f(x, uy_1(x)),$$

which is equivalent to

$$u'y_1(x) = f(x, uy_1(x)) - uy_1'(x). \tag{2.4.5}$$

If

$$f(x, uy_1(x)) = q(u)y_1'(x)$$

for some function q, then (2.4.5) becomes

$$u'y_1(x) = (q(u) - u)y_1'(x), \tag{2.4.6}$$

which is separable. After checking for constant solutions $u \equiv u_0$ such that $q(u_0) = u_0$, we can separate variables to obtain

$$\frac{u'}{q(u) - u} = \frac{y_1'(x)}{y_1(x)}.$$

Homogeneous Nonlinear Equations

In the text we'll consider only the most widely studied class of equations for which the method of the preceding paragraph works. Other types of equations appear in Exercises 44–51.

The differential equation (2.4.4) is said to be *homogeneous* if x and y occur in f in such a way that $f(x, y)$ depends only on the ratio y/x; that is, (2.4.4) can be written as

$$y' = q(y/x), \tag{2.4.7}$$

where $q = q(u)$ is a function of a single variable. For example,

$$y' = \frac{y + xe^{-y/x}}{x} = \frac{y}{x} + e^{-y/x}$$

and

$$y' = \frac{y^2 + xy - x^2}{x^2} = \left(\frac{y}{x}\right)^2 + \frac{y}{x} - 1$$

are of the form (2.4.7), with

$$q(u) = u + e^{-u} \quad \text{and} \quad q(u) = u^2 + u - 1,$$

respectively. The general method discussed above can be applied to (2.4.7) with $y_1 = x$ (and therefore $y_1' = 1$). Thus, substituting $y = ux$ in (2.4.7) yields

$$u'x + u = q(u),$$

and separation of variables (after checking for constant solutions $u \equiv u_0$ such that $q(u_0) = u_0$) yields

$$\frac{u'}{q(u) - u} = \frac{1}{x}.$$

Before turning to examples, we point out something that you may've have already noticed: the definition of *homogeneous equation* given here isn't the same as the definition given in Section 2.1, where we said that a linear equation of the form

$$y' + p(x)y = 0$$

is homogeneous. We make no apology for this inconsistency, since we didn't create it historically, *homogeneous* has been used in these two inconsistent ways. The one having to do with linear equations is the most important. This is the only section of the book where the meaning defined here will apply.

Since y/x is in general undefined if $x = 0$, we'll consider solutions of nonhomogeneous equations only on open intervals that do not contain the point $x = 0$.

Example 2.4.2 Solve

$$y' = \frac{y + xe^{-y/x}}{x}. \tag{2.4.8}$$

Solution Substituting $y = ux$ into (2.4.8) yields

$$u'x + u = \frac{ux + xe^{-ux/x}}{x} = u + e^{-u}.$$

Simplifying and separating variables yields

$$e^u u' = \frac{1}{x}.$$

Integrating yields $e^u = \ln|x| + c$. Therefore $u = \ln(\ln|x| + c)$ and $y = ux = x\ln(\ln|x| + c)$.
Figure 2.4.2 shows a direction field and integral curves for (2.4.8).

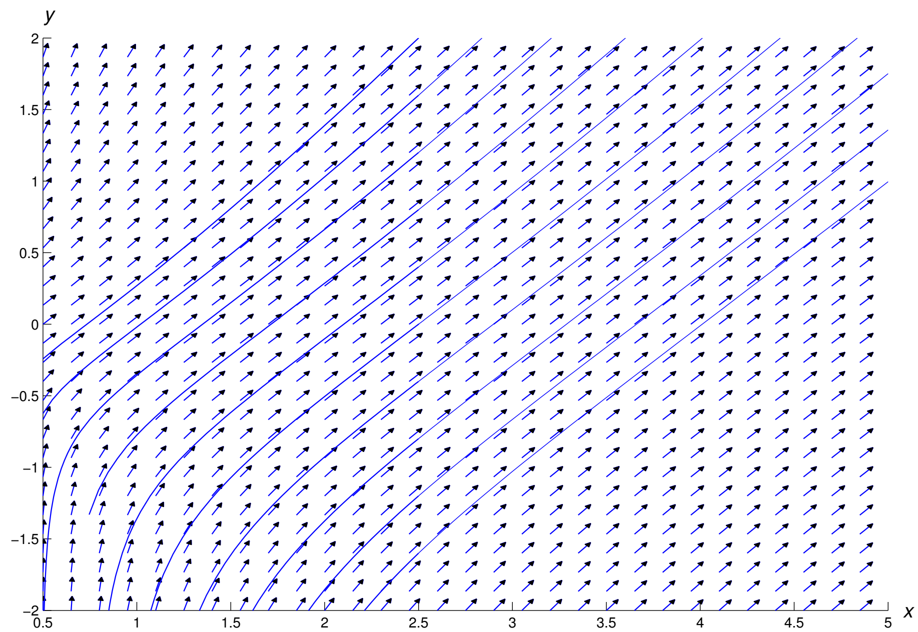

Figure 2.4.2 A direction field and some integral curves for $y' = \dfrac{y + xe^{-y/x}}{x}$

Example 2.4.3

(a) Solve

$$x^2y' = y^2 + xy - x^2. \tag{2.4.9}$$

(b) Solve the initial value problem

$$x^2y' = y^2 + xy - x^2, \quad y(1) = 2. \tag{2.4.10}$$

SOLUTION(**a**) We first find solutions of (2.4.9) on open intervals that don't contain $x = 0$. We can rewrite (2.4.9) as

$$y' = \frac{y^2 + xy - x^2}{x^2}$$

for x in any such interval. Substituting $y = ux$ yields

$$u'x + u = \frac{(ux)^2 + x(ux) - x^2}{x^2} = u^2 + u - 1,$$

so

$$u'x = u^2 - 1. \tag{2.4.11}$$

By inspection this equation has the constant solutions $u \equiv 1$ and $u \equiv -1$. Therefore $y = x$ and $y = -x$ are solutions of (2.4.9). If u is a solution of (2.4.11) that doesn't assume the values ± 1 on some interval, separating variables yields

$$\frac{u'}{u^2-1} = \frac{1}{x},$$

or, after a partial fraction expansion,

$$\frac{1}{2}\left[\frac{1}{u-1} - \frac{1}{u+1}\right]u' = \frac{1}{x}.$$

Multiplying by 2 and integrating yields

$$\ln\left|\frac{u-1}{u+1}\right| = 2\ln|x| + k,$$

or

$$\left|\frac{u-1}{u+1}\right| = e^k x^2,$$

which holds if

$$\frac{u-1}{u+1} = cx^2 \tag{2.4.12}$$

where c is an arbitrary constant. Solving for u yields

$$u = \frac{1+cx^2}{1-cx^2}.$$

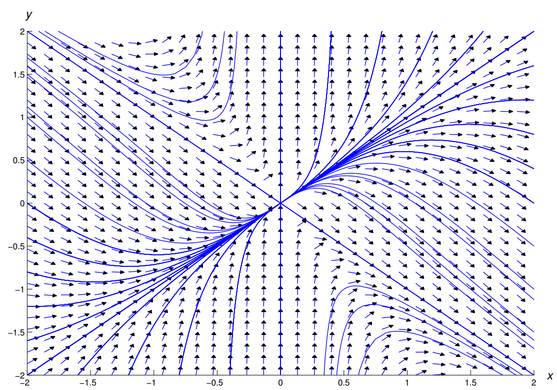

Figure 2.4.3 A direction field and integral curves for $x^2y' = y^2 + xy - x^2$

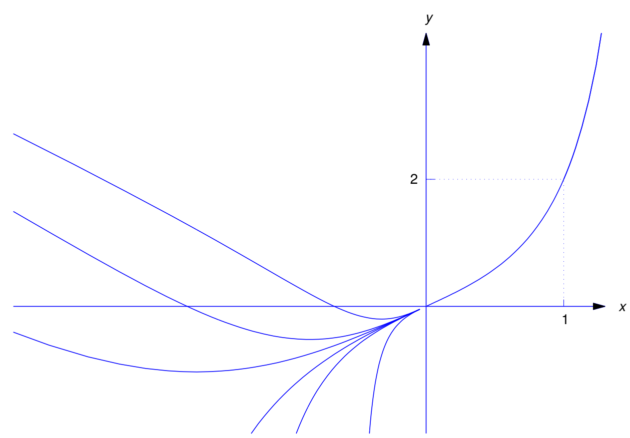

Figure 2.4.4 Solutions of $x^2y' = y^2 + xy - x^2$, $y(1) = 2$

Therefore

$$y = ux = \frac{x(1+cx^2)}{1-cx^2} \tag{2.4.13}$$

is a solution of (2.4.10) for any choice of the constant c. Setting $c = 0$ in (2.4.13) yields the solution $y = x$. However, the solution $y = -x$ can't be obtained from (2.4.13). Thus, the solutions of (2.4.9) on intervals that don't contain $x = 0$ are $y = -x$ and functions of the form (2.4.13).

The situation is more complicated if $x = 0$ is the open interval. First, note that $y = -x$ satisfies (2.4.9) on $(-\infty, \infty)$. If c_1 and c_2 are arbitrary constants, the function

$$y = \begin{cases} \dfrac{x(1 + c_1x^2)}{1 - c_1x^2}, & a < x < 0, \\ \dfrac{x(1 + c_2x^2)}{1 - c_2x^2}, & 0 \le x < b, \end{cases} \tag{2.4.14}$$

is a solution of (2.4.9) on (a, b), where

$$a = \begin{cases} -\dfrac{1}{\sqrt{c_1}} & \text{if } c_1 > 0, \\ -\infty & \text{if } c_1 \le 0, \end{cases} \quad \text{and} \quad b = \begin{cases} \dfrac{1}{\sqrt{c_2}} & \text{if } c_2 > 0, \\ \infty & \text{if } c_2 \le 0. \end{cases}$$

We leave it to you to verify this. To do so, note that if y is any function of the form (2.4.13) then $y(0) = 0$ and $y'(0) = 1$.

Figure 2.4.3 shows a direction field and some integral curves for (2.4.9).

SOLUTION(**b**) We could obtain c by imposing the initial condition $y(1) = 2$ in (2.4.13), and then solving for c. However, it's easier to use (2.4.12). Since $u = y/x$, the initial condition $y(1) = 2$ implies that $u(1) = 2$. Substituting this into (2.4.12) yields $c = 1/3$. Hence, the solution of (2.4.10) is

$$y = \frac{x(1 + x^2/3)}{1 - x^2/3}.$$

The interval of validity of this solution is $(-\sqrt{3}, \sqrt{3})$. However, the largest interval on which (2.4.10) has a unique solution is $(0, \sqrt{3})$. To see this, note from (2.4.14) that any function of the form

$$y = \begin{cases} \dfrac{x(1 + cx^2)}{1 - cx^2}, & a < x \le 0, \\ \dfrac{x(1 + x^2/3)}{1 - x^2/3}, & 0 \le x < \sqrt{3}, \end{cases} \tag{2.4.15}$$

is a solution of (2.4.10) on $(a, \sqrt{3})$, where $a = -1/\sqrt{c}$ if $c > 0$ or $a = -\infty$ if $c \le 0$. (Why doesn't this contradict Theorem 2.3.1?)

Figure 2.4.4 shows several solutions of the initial value problem (2.4.10). Note that these solutions coincide on $(0, \sqrt{3})$.

In the last two examples we were able to solve the given equations explicitly. However, this isn't always possible, as you'll see in the exercises.

2.4 Exercises

In Exercises 1–4 solve the given Bernoulli equation.

1. $y' + y = y^2$

2. $7xy' - 2y = -\dfrac{x^2}{y^6}$

3. $x^2y' + 2y = 2e^{1/x}y^{1/2}$

4. $(1 + x^2)y' + 2xy = \dfrac{1}{(1 + x^2)y}$

In Exercises 5 and 6 find all solutions. Also, plot a direction field and some integral curves on the indicated rectangular region.

5. C/G $y' - xy = x^3y^3$; $\{-3 \le x \le 3, 2 \le y \ge 2\}$

6. C/G $y' - \dfrac{1+x}{3x}y = y^4$; $\{-2 \le x \le 2, -2 \le y \le 2\}$

In Exercises 7–11 solve the initial value problem.

7. $y' - 2y = xy^3, \quad y(0) = 2\sqrt{2}$

8. $y' - xy = xy^{3/2}, \quad y(1) = 4$

9. $xy' + y = x^4y^4, \quad y(1) = 1/2$

10. $y' - 2y = 2y^{1/2}, \quad y(0) = 1$

11. $y' - 4y = \dfrac{48x}{y^2}, \quad y(0) = 1$

In Exercises 12 and 13 solve the initial value problem and graph the solution.

12. C/G $x^2y' + 2xy = y^3, \quad y(1) = 1/\sqrt{2}$

13. C/G $y' - y = xy^{1/2}, \quad y(0) = 4$

14. You may have noticed that the logistic equation

$$P' = aP(1 - \alpha P)$$

from Verhulst's model for population growth can be written in Bernoulli form as

$$P' - aP = -a\alpha P^2.$$

This isn't particularly interesting, since the logistic equation is separable, and therefore solvable by the method studied in Section 2.2. So let's consider a more complicated model, where a is a positive constant and α is a positive continuous function of t on $[0, \infty)$. The equation for this model is

$$P' - aP = -a\alpha(t)P^2,$$

a non-separable Bernoulli equation.

(a) Assuming that $P(0) = P_0 > 0$, find P for $t > 0$. HINT: *Express your result in terms of the integral* $\int_0^t \alpha(\tau)e^{a\tau}\,d\tau$.

(b) Verify that your result reduces to the known results for the Malthusian model where $\alpha = 0$, and the Verhulst model where α is a nonzero constant.

(c) Assuming that

$$\lim_{t\to\infty} e^{-at}\int_0^t \alpha(\tau)e^{a\tau}\,d\tau = L$$

exists (finite or infinite), find $\lim_{t\to\infty} P(t)$.

In Exercises 15–18 solve the equation explicitly.

15. $y' = \dfrac{y+x}{x}$

16. $y' = \dfrac{y^2 + 2xy}{x^2}$

17. $xy^3y' = y^4 + x^4$

18. $y' = \dfrac{y}{x} + \sec\dfrac{y}{x}$

In Exercises 19-21 solve the equation explicitly. Also, plot a direction field and some integral curves on the indicated rectangular region.

19. C/G $x^2y' = xy + x^2 + y^2; \quad \{-8 \le x \le 8, -8 \le y \le 8\}$

20. C/G $xyy' = x^2 + 2y^2; \quad \{-4 \le x \le 4, -4 \le y \le 4\}$

21. C/G $y' = \dfrac{2y^2 + x^2e^{-(y/x)^2}}{2xy}; \quad \{-8 \le x \le 8, -8 \le y \le 8\}$

In Exercises 22–27 solve the initial value problem.

22. $y' = \dfrac{xy + y^2}{x^2}, \quad y(-1) = 2$

23. $y' = \dfrac{x^3 + y^3}{xy^2}, \quad y(1) = 3$

24. $xyy' + x^2 + y^2 = 0, \quad y(1) = 2$

25. $y' = \dfrac{y^2 - 3xy - 5x^2}{x^2}, \quad y(1) = -1$

26. $x^2y' = 2x^2 + y^2 + 4xy, \quad y(1) = 1$

27. $xyy' = 3x^2 + 4y^2, \quad y(1) = \sqrt{3}$

In Exercises 28–34 solve the given homogeneous equation implicitly.

28. $y' = \dfrac{x + y}{x - y}$

29. $(y'x - y)(\ln|y| - \ln|x|) = x$

30. $y' = \dfrac{y^3 + 2xy^2 + x^2y + x^3}{x(y + x)^2}$

31. $y' = \dfrac{x + 2y}{2x + y}$

32. $y' = \dfrac{y}{y - 2x}$

33. $y' = \dfrac{xy^2 + 2y^3}{x^3 + x^2y + xy^2}$

34. $y' = \dfrac{x^3 + x^2y + 3y^3}{x^3 + 3xy^2}$

35. L

(a) Find a solution of the initial value problem

$$x^2y' = y^2 + xy - 4x^2, \quad y(-1) = 0 \tag{A}$$

on the interval $(-\infty, 0)$. Verify that this solution is actually valid on $(-\infty, \infty)$.

(b) Use Theorem 2.3.1 to show that (A) has a unique solution on $(-\infty, 0)$.

(c) Plot a direction field for the differential equation in (A) on a square

$$\{-r \le x \le r, -r \le y \le r\},$$

where r is any positive number. Graph the solution you obtained in **(a)** on this field.

(d) Graph other solutions of (A) that are defined on $(-\infty, \infty)$.

(e) Graph other solutions of (A) that are defined only on intervals of the form $(-\infty, a)$, where is a finite positive number.

36. L

(a) Solve the equation

$$xyy' = x^2 - xy + y^2 \tag{A}$$

implicitly.

(b) Plot a direction field for (A) on a square

$$\{0 \le x \le r, 0 \le y \le r\}$$

where r is any positive number.

(c) Let K be a positive integer. (You may have to try several choices for K.) Graph solutions of the initial value problems

$$xyy' = x^2 - xy + y^2, \quad y(r/2) = \frac{kr}{K},$$

for $k = 1, 2, \dots, K$. Based on your observations, find conditions on the positive numbers x_0 and y_0 such that the initial value problem

$$xyy' = x^2 - xy + y^2, \quad y(x_0) = y_0, \tag{B}$$

has a unique solution (i) on $(0, \infty)$ or (ii) only on an interval (a, ∞), where $a > 0$?

(d) What can you say about the graph of the solution of (B) as $x \to \infty$? (Again, assume that $x_0 > 0$ and $y_0 > 0$.)

37. L

(a) Solve the equation

$$y' = \frac{2y^2 - xy + 2x^2}{xy + 2x^2} \tag{A}$$

implicitly.

(b) Plot a direction field for (A) on a square

$$\{-r \le x \le r, -r \le y \le r\}$$

where r is any positive number. By graphing solutions of (A), determine necessary and sufficient conditions on (x_0, y_0) such that (A) has a solution on (i) $(-\infty, 0)$ or (ii) $(0, \infty)$ such that $y(x_0) = y_0$.

38. L Follow the instructions of Exercise 37 for the equation

$$y' = \frac{xy + x^2 + y^2}{xy}.$$

39. L Pick any nonlinear homogeneous equation $y' = q(y/x)$ you like, and plot direction fields on the square $\{-r \le x \le r, \ -r \le y \le r\}$, where $r > 0$. What happens to the direction field as you vary r? Why?

40. Prove: If $ad - bc \neq 0$, the equation

$$y' = \frac{ax + by + \alpha}{cx + dy + \beta}$$

can be transformed into the homogeneous nonlinear equation

$$\frac{dY}{dX} = \frac{aX + bY}{cX + dY}$$

by the substitution $x = X - X_0,\ y = Y - Y_0$, where X_0 and Y_0 are suitably chosen constants.

In Exercises 41-43 use a method suggested by Exercise 40 to solve the given equation implicitly.

41. $y' = \dfrac{-6x + y - 3}{2x - y - 1}$

42. $y' = \dfrac{2x + y + 1}{x + 2y - 4}$

43. $y' = \dfrac{-x + 3y - 14}{x + y - 2}$

In Exercises 44–51 find a function y_1 such that the substitution $y = uy_1$ transforms the given equation into a separable equation of the form (2.4.6). Then solve the given equation explicitly.

44. $3xy^2y' = y^3 + x$

45. $xyy' = 3x^6 + 6y^2$

46. $x^3y' = 2(y^2 + x^2y - x^4)$

47. $y' = y^2e^{-x} + 4y + 2e^x$

48. $y' = \dfrac{y^2 + y\tan x + \tan^2 x}{\sin^2 x}$

49. $x(\ln x)^2y' = -4(\ln x)^2 + y\ln x + y^2$

50. $2x(y + 2\sqrt{x})y' = (y + \sqrt{x})^2$

51. $(y + e^{x^2})y' = 2x(y^2 + ye^{x^2} + e^{2x^2})$

52. Solve the initial value problem

$$y' + \frac{2}{x}y = \frac{3x^2y^2 + 6xy + 2}{x^2(2xy + 3)}, \quad y(2) = 2.$$

53. Solve the initial value problem

$$y' + \frac{3}{x}y = \frac{3x^4y^2 + 10x^2y + 6}{x^3(2x^2y + 5)}, \quad y(1) = 1.$$

54. Prove: If y is a solution of a homogeneous nonlinear equation $y' = q(y/x)$, so is $y_1 = y(ax)/a$, where a is any nonzero constant.

55. A *generalized Riccati equation* is of the form

$$y' = P(x) + Q(x)y + R(x)y^2. \tag{A}$$

(If $R \equiv -1$, (A) is a *Riccati equation*.) Let y_1 be a known solution and y an arbitrary solution of (A). Let $z = y - y_1$. Show that z is a solution of a Bernoulli equation with $n = 2$.

In Exercises 56–59, given that y_1 is a solution of the given equation, use the method suggested by Exercise 55 to find other solutions.

56. $y' = 1 + x - (1+2x)y + xy^2; \quad y_1 = 1$

57. $y' = e^{2x} + (1-2e^x)y + y^2; \quad y_1 = e^x$

58. $xy' = 2 - x + (2x-2)y - xy^2; \quad y_1 = 1$

59. $xy' = x^3 + (1-2x^2)y + xy^2; \quad y_1 = x$

2.5 EXACT EQUATIONS

In this section it's convenient to write first order differential equations in the form

$$M(x, y)\,dx + N(x, y)\,dy = 0. \tag{2.5.1}$$

This equation can be interpreted as

$$M(x, y) + N(x, y)\,\frac{dy}{dx} = 0, \tag{2.5.2}$$

where x is the independent variable and y is the dependent variable, or as

$$M(x, y)\,\frac{dx}{dy} + N(x, y) = 0, \tag{2.5.3}$$

where y is the independent variable and x is the dependent variable. Since the solutions of (2.5.2) and (2.5.3) will often have to be left in implicit, form we'll say that $F(x, y) = c$ is an implicit solution of (2.5.1) if every differentiable function $y = y(x)$ that satisfies $F(x, y) = c$ is a solution of (2.5.2) and every differentiable function $x = x(y)$ that satisfies $F(x, y) = c$ is a solution of (2.5.3).

Here are some examples:

Equation (2.5.1)	Equation (2.5.2)	Equation (2.5.3)
$3x^2y^2\,dx + 2x^3y\,dy = 0$	$3x^2y^2 + 2x^3y\,\dfrac{dy}{dx} = 0$	$3x^2y^2\,\dfrac{dx}{dy} + 2x^3y = 0$
$(x^2 + y^2)\,dx + 2xy\,dy = 0$	$(x^2 + y^2) + 2xy\,\dfrac{dy}{dx} = 0$	$(x^2 + y^2)\,\dfrac{dx}{dy} + 2xy = 0$
$3y\sin x\,dx - 2xy\cos x\,dy = 0$	$3y\sin x - 2xy\cos x\,\dfrac{dy}{dx} = 0$	$3y\sin x\,\dfrac{dx}{dy} - 2xy\cos x = 0$

Note that a separable equation can be written as (2.5.1) as

$$M(x)\,dx + N(y)\,dy = 0.$$

We'll develop a method for solving (2.5.1) under appropriate assumptions on M and N. This method is an extension of the method of separation of variables (Exercise 41). Before stating it we consider an example.