Math 260 **Models with Differential Equations** **January 29, 2018**

1. We saw that the exponential model for population growth $P' = aP$ predicts unbounded population sizes. The logistic model resolves this problem: $P' = aP(1 - \alpha P)$ where a and α are positive constants.

a) Draw a phase line for the model and use it to predict the long-term population trends.

b) Find a solution to the differential equation using separation of variables.

c) Calculate $\lim_{t\to\infty} P(t)$ and compare with your answer for part a.

2. Newton's law of cooling states that the rate of change of an object's temperature is proportional to the difference between its temperature and the temperature of its environment. As a differential equation: $T' = -k(T - T_m)$ where k is a positive constant of proportionality and T_m is the (constant) temperature of the environment.

a) Draw a phase line and use it to predict the eventual temperature of the object.

b) Find the general solution to the differential equation.

3. An object with initial temperature $150°$C is placed outside, where the temperature is $10°$C. Its temperatures at 12:15 and 12:20 are $120°$C and $90°$C, respectively. At what time was the object placed outside?

4. A 960 lb object is launched upward with initial velocity 60 ft/s. The atmosphere resists the object's motion with a force of 3 lb for each ft/s of speed. Assume that the only other force acting on the object is gravity (the acceleration of which is 32 ft/s^2 downward). Find the terminal velocity of the object.