Theorem. If y_p is any particular solution to the differential equation

$$y'' + p(x)y' + q(x)y = f(x) \tag{1}$$

and $\{y_1, y_2\}$ is a fundamental set of solutions to the complementary equation $y'' + p(x)y' + q(x)y = 0$, then a general solution for differential equation 1 is

$$y = y_p + c_1y_1 + c_2y_2$$

Theorem (Superposition)**.** If y_{p_1} is a particular solution of $y'' + p(x)y' + q(x)y = f_1(x)$ and y_{p_2} is a particular solution of $y'' + p(x)y' + q(x)y = f_2(x)$, then a particular solution of

$$y'' + p(x)y' + q(x)y = f_1(x) + f_2(x)$$

is

$$y_p = y_{p_1} + y_{p_2}$$

1. Consider the differential equation $y'' - 7y' + 12y = 4e^{2x}$.

a) Find a constant A such that $y_p = Ae^{2x}$ is a solution of the differential equation.

b) Find a general solution for the differential equation.

2. Consider the differential equation $y'' - 7y' + 12y = 5e^{4x}$.

a) Why can't you find a constant A such that $y_p = Ae^{4x}$ is a solution of the differential equation?

b) Find a constant B such that Bxe^{4x} is a solution of the differential equation.

c) Find a general solution for the differential equation.

3. Use the principle of superposition to find a general solution to the differential equation $y'' - 7y' + 12y = 5e^{4x} + 4e^{2x}$.

4. Consider the differential equation $y'' - 8y' + 16y = 2e^{4x}$.

a) Why can't you find a constant A such that $y_p = Ae^{4x}$ is a solution of the differential equation?

b) Why can't you find a constant B such that Bxe^{4x} is a solution of the differential equation?

c) Find a constant C such that Cx^2e^{4x} is a solution of the differential equation.

d) Find a general solution for the differential equation.