1. Goal: use Laplace transforms to solve the IVP: $y'' - 2y' = \begin{cases} 4, & 0 \le t < 1 \\ 6, & t \ge 1 \end{cases}$, $y(0) = -6$, $y'(0) = 1$.

a) Express $f(t) = \begin{cases} 4, & 0 \le t < 1 \\ 6, & t \ge 1 \end{cases}$ as $f(t) = f_0(t) + u(t - t_1)\left[f_1(t) - f_0(t)\right]$ for some functions f_0 and f_1 and a constant t_1.

b) Use your solution for part a to find the Laplace transform of $f(t)$. Use version 1 of the second shifting theorem: $L\left(u(t - t_1)g(t)\right) = e^{-t_1 s} L\left(g(t + t_1)\right)$.

c) Take the Laplace transform of the entire differential equation.

d) Sub in $L(y'') = s^2 L(y) - sy(0) - y'(0)$ and $L(y') = sL(y) - y(0)$ and solve for $Y(s) = L(y)$.

e) Take the inverse Laplace transform to get the solution $y(t) = L^{-1}(Y)$. Use the second shifting theorem: $L\left(u(t - t_1)g(t - t_1)\right) = e^{-t_1 s} L(g)$.

Definition. Let f and g be functions such that if $t < 0$, then $f(t) = g(t) = 0$. The **convolution** of f and g is the function $f * g$ defined by

$$(f * g)(t) = \int_0^t f(\tau)g(t-\tau)d\tau$$

Theorem (Convolution Theorem)**.**

$$L(f * g) = L(f)L(g)$$

2. Use the Convolution Theorem to evaluate the integral $\int_0^2 (2-\tau)^5 \tau^7 d\tau$ by:

a) Identifying $h(t) = \int_0^t (t-\tau)^5 \tau^7 d\tau$ as the convolution of two functions f and g.

b) Applying the Convolution Theorem to find $L(h)$.

c) Taking the inverse Laplace transform to find $h(t)$.

d) $\int_0^2 (2-\tau)^5 \tau^7 d\tau = h(2) =$?