Sometimes a small change in a parameter in an ODE in can have a big effect on the behavior of the solutions. This is called a **bifurcation event**. We will study the bifurcations that occur in the system

$$\boxed{\mathbf{y}' = \begin{pmatrix} \alpha & 1 \\ -4 & 0 \end{pmatrix} \mathbf{y}}$$

1. Download and run pplane.jar (`http://math.rice.edu/~dfield/dfpp.html`). Set the Display Window so that both x and y range between -2 and 2. Graph the phase plane for $\alpha = -5$, $\alpha = 0$, $\alpha = 2$, $\alpha = 4$, and $\alpha = 8$. (Hint: for $\alpha = 8$ you'll need to enter the system of equations $x' = 8x + y$ and $y' = -4x$). Compare and contrast the phase plane graphs.

2. Find the eigenvalues of $\begin{pmatrix} \alpha & 1 \\ -4 & 0 \end{pmatrix}$ as a function of α. (Hint: use the quadratic formula).

a) For what values of α does the matrix have distinct real eigenvalues?

b) For what values of α does the matrix have repeated real eigenvalues?

c) For what values of α does the matrix have imaginary eigenvalues?

3. Choose any value of α you want and solve the IVP (use the methods we have been discussing in class, as summarized in this linked document; note that solutions are hard to find for some choices of α):

$$\mathbf{y}' = \begin{pmatrix} \alpha & 1 \\ -4 & 0 \end{pmatrix} \mathbf{y}, \ \mathbf{y}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

4. Use pplane.jar to plot the solution to your IVP by entering the correct value of α and selecting "Keyboard Input of Initial Value" from the Solution menu and entering $x = 1$ and $y = 0$. Sketch the graph along with dashed lines for the eigenvector(s) of the matrix (if real-valued).

5. Check the "Use current initial values in new graph" box at the bottom left of the equation window of pplane.jar and then graph the phase plane for $\alpha = -5$, $\alpha = 0$, $\alpha = 2$, $\alpha = 4$, and $\alpha = 8$ (this time you should see the graph of the solution to the IVP in addition to the arrows). Compare these graphs with your (and admire the spirals).