Math 260 **Exam 1** **February 7, 2018**

1. Find the general solution to the differential equation $e^x y' + y = 0$.

2. Solve the initial value problem $y' - y = 2e^x$, $y(1) = 3e$.

3. Solve the initial value problem $y' = \dfrac{2x}{1+2y}$, $y(2) = 0$. An implicit solution is sufficient: you do not need to solve for y.

4. Determine if $y = \sqrt{4 - x^2}$ is a solution to the differential equation $yy' + x = 0$. Your answer must be justified by your calculations.

5. Find all equilibrium solutions to the differential equation $y' = y^2(1 - y^2)$ and identify each equilibrium as stable, semi-stable, or unstable.

6. Suppose that a mass has been attached to a carefully engineered "progressive" spring and as a result it's position $y = y(t)$ satisfies the autonomous second-order differential equation $y'' + y^3 = 0$. Find a phase plane solution to the differential equation (implicit solutions are preferred).

7. A population grows logistically according to the equation $P' = \frac{3}{50}P(1 - \frac{1}{500}P)$ and $P(0) > 0$. Find $\lim_{t\to\infty} P(t)$. Hint: there is no need to solve the differential equation.