Math 260 **Exam 1** **February 7, 2018**

Instructions: Solve 6 of the following 7 problems and write your solutions on the provided paper, clearly labeling each solution (do not write your solutions on this sheet). All solutions should include a clear method or argument and should use English words and sentences when appropriate. Clear and comprehensible solutions will generally earn more points than those that are hard to understand; a correct solution without supporting work may receive little or no credit. Indicate which problem you are skipping by placing an **X** in the corresponding box below. Leave the rest blank (I'll use them to record your scores).

Calculators, phones, and all other devices are forbidden. Answers may be left unsimplified.

Name:							
1	2	3	4	5	6	7	Total

Thoerem. *If the characteristic polynomial of $ay'' + by' + cy' = 0$ has...*

a) ...distinct real roots r_1 and r_2, then a general solution is $y = c_1e^{r_1x} + c_2e^{r_2x}$

b) ...a single (repeated) real root r, then a general solution is $y = e^{rx}(c_1 + c_2x)$

c) ...complex conjugate roots $\lambda \pm i\omega$ (where $\omega > 0$), then a general solution is $y = e^{\lambda x}(c_1 \cos \omega x + c_2 \sin \omega x)$

Thoerem. *If y_p is any particular solution to the differential equation $y'' + p(x)y' + q(x)y = f(x)$ and $\{y_1, y_2\}$ is a fundamental set of solutions to the complementary equation, then a general solution for differential equation is*

$$y = y_p + c_1y_1 + c_2y_2$$

Thoerem (Superposition)**.** *If y_{p_1} is a particular solution of $y'' + p(x)y' + q(x)y = f_1(x)$ and y_{p_2} is a particular solution of $y'' + p(x)y' + q(x)y = f_2(x)$, then a particular solution of*

$$y'' + p(x)y' + q(x)y = f_1(x) + f_2(x)$$

is

$$y_p = y_{p_1} + y_{p_2}$$

Method. To find a particular solution for $ay'' + by' + cy = e^{\alpha x}G(x)$ (where G is a polynomial), solve for the coefficients of $Q(x)$ in the following (where Q is a polynomial with the same degree as G):

a) $y_p = e^{\alpha x}Q(x)$ if $e^{\alpha x}$ is not a solution to the complementary equation;

b) $y_p = xe^{\alpha x}Q(x)$ if $xe^{\alpha x}$ is not a solution to the complementary equation, but $e^{\alpha x}$ is a solution to the complementary equation;

c) $y_p = x^2e^{\alpha x}Q(x)$ if $xe^{\alpha x}$ and $e^{\alpha x}$ are both solutions to the complementary equation.

Method. To find a particular solution for $ay'' + by' + cy = P(x) \cos \omega x + Q(x) \sin \omega x$ (where P and Q are polynomials), solve for the coefficients of $A(x)$ and $B(x)$ in the following (where A and B are polynomials with degree equal to the larger of the degrees of P and Q):

a) $y_p = A(x) \cos \omega x + B(x) \sin \omega x$ if $\cos \omega x$ and $\sin \omega x$ are not solutions to the complementary equation;

b) $y_p = x\left[A(x) \cos \omega x + B(x) \sin \omega x\right]$ if $\cos \omega x$ and $\sin \omega x$ are solutions to the complementary equation;

1. Determine which one of the following pairs of functions y_1 and y_2 forms a fundamental set of solutions to the differential equation $x^2y'' + xy' - 4y = 0$ on the interval $(0, \infty)$. You may use the Wronskian (but you are not required to use the Wronskian): $W(y_1, y_2) = y_1y_2' - y_1'y_2$.

a) $y_1 = x^2$ and $y_2 = x$

b) $y_1 = x^2$ and $y_2 = x^{-2}$

c) $y_1 = x^2$ and $y_2 = -2x^2$

2. Solve the initial value problem: $y'' - 4y' + 3y = 0$, $y(0) = 2$, $y'(0) = 0$.

3. Find a number A so that $y_p = Ax^4$ is a solution of $x^2y'' - 3xy' + 13y = 2x^4$.

4. Find a particular solution of $y'' - 3y' - 10y = 6e^{5t}$.

5. A 4 kg mass is attached to a spring with a spring constant of $k = 1$ and a damping constant of $c = 4$. Find a general solution for the motion of the spring (your answer should not be a differential equation; it should have 2 unknown constants since I have not provided you with initial conditions).

6. An object stretches a spring 0.5 m at equilibrium. An oscillating external force of $5\sin 4t$ N is applied. Determine if the natural frequency of the spring-mass system is greater than, less than, or equal to the frequency of the applied force. Use $g = 9.8$ m/s^2 for the acceleration of gravity.

7. Find $a_0, a_1, a_2, a_3,$ and a_4 in the power series $y = \sum_{n=0}^{\infty} a_nx^n$ for the solution to the initial value problem $(x^2 - 1)y'' + y = 0$, $y(0) = -2$, $y'(0) = 0$.