

## LINEAR DIFFERENTIAL EQUATIONS

**Theorem.** The **general solution** to the homogeneous linear differential equation  $y' + p(x)y = 0$  is

$$y = ce^{-P(x)}$$

where  $P(x)$  is any antiderivative of  $p(x)$  (any function so that  $P'(x) = p(x)$ ).

Note that applying the theorem requires that  $p(x)$  be integrable; thus there are situations in which the theorem doesn't help.

1. This problem deals with the differential equation  $(1 + x^2)y' = 2xy$ .

a) Find the general solution

b) Check that your solution really works

c) Find the particular solution with  $y(1) = 4$ .

**Theorem.** The general solution to the linear differential equation  $y' + p(x)y = f(x)$  is

$$y = uy_1$$

where

a)  $y_1$  is any particular solution to the complementary equation  $y' + p(x)y = 0$

b)  $u = \int \frac{f(x)}{y_1(x)} dx$  (add a constant here).

2. Solve the IVP:  $y' + 2xy = x$ ,  $y(1) = 1$ .