MODELS USING DIFFERENTIAL EQUATIONS

1. We saw that the exponential model for population growth $P' = aP$ predicts unbounded population sizes. The logistic model resolves this problem: $P' = aP(1 - bP)$ where a and b are positive constants. If it helps you to solve the problem, you may use $a = 1$ and $b = 0.5$.

a) Draw a phase line and use it to predict the long-term population trends.

b) Use separation of variables to find a solution $P(t)$. (You may need to use the method of partial fractions).

c) Calculate $\lim_{t\to\infty} P(t)$ and compare with your answer for part a.

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2. Newton's law of cooling states that the rate of change of an object's temperature is proportional to the difference between its temperature and the temperature of its environment. As a differential equation: $T' = -k(T - T_m)$ where k is a positive constant of proportionality and T_m is the (constant) temperature of the environment.

a) What is the eventual temperature T of the object?

b) Draw a phase line for the model and verify that it agrees with your answer for part a.

c) Find the general solution to the differential equation.

3 (CSI Gonzaga)**.** A body is found outside on a $5°$C day. At 12:15 its temperature is $35°$C and at 12:45 its temperature is $33°$C.

a) Use the two points to find a formula for $T(t)$, the temperature of the body t hours after 12:15.

b) Human body temperature is $37°$C. Use this to find the time the body started cooling.

Bonus. A 10kg object is launched upward with initial velocity 60 m/s. The atmosphere resists the object's motion with a force of 5 N for each m/s of speed. Assume that the only other force acting on the object is gravity (the acceleration of which is 9.8 m/s^2 downward). Easier: find the terminal velocity of the object. Harder: Find a formula for the velocity of the object.