

Method. To find a solution y_p to the differential equation $ay'' + by' + cy = P(x)$ where $P(x)$ is a polynomial of degree n (and $c \neq 0$): set $y_p = A_n x^n + A_{n-1} x^{n-1} + \cdots + A_0$ and solve for the undetermined coefficients A_0, A_1, \dots, A_n .

1. Find a general solution to $y'' - 2y' + y = x - 3$.

Method. To find a solution y_p to the differential equation $ay'' + by' + cy = ke^{\alpha x}$: set y_p equal to the first of the following that is not a solution to the complementary equation and solve for the undetermined coefficients.

1. $y_p = Ae^{\alpha x}$

2. $y_p = Axe^{\alpha x}$

3. $y_p = Ax^2e^{\alpha x}$

2. Find a general solution to $y'' - 3y' + 2y = e^x$.

Method. To find a solution y_p to the differential equation $ay'' + by' + cy = p \cos(\omega x) + q \sin(\omega x)$: set y_p equal to the first of the following that is not a solution to the complementary equation and solve for the undetermined coefficients.

1. $y_p = A \cos(\omega x) + B \sin(\omega x)$

2. $y_p = Ax \cos(\omega x) + Bx \sin(\omega x)$

3. A spring system with negligible friction and cyclic external force satisfies the differential equation $y'' + y = 8 \cos t$. Solve the IVP if $y(0) = 0$ and $y'(0) = 0$.