Math 260 **Laplace transforms** **March 12, 2020**

Video 1. Start with the video introducing Laplace transforms.

1. Find the Laplace transform of $f(t) = 1$

2. Find the Laplace transform of $f(t) = e^{2t}$

3. Find the Laplace transform of $f(t) = 1 + 2t + 3e^{2t}$

Video 2. Watch the next video once you're stuck or done.

Theorem (Linearity Property)**.** Let $c_1, c_2, \ldots, c_n$ be constants and let $f_1, f_2, \ldots, f_n$ be functions. Suppose the Laplace transform $L(f_i)$ is defined for $s > s_i$ for each i and let s_0 be the largest of $s_1, s_2, \ldots, s_n$. Then

$$L(c_1 f_1 + c_2 f_2 + \cdots + c_n f_n) = c_1 L(f_1) + c_2 L(f_2) + \cdots + c_n L(f_n) \text{ for } s > s_0$$

Theorem (First Shifting Theorem)**.** If $L\left(f(t)\right) = F(s)$ for $s > s_0$, then

$$L\left(e^{at} f(t)\right) = F(s-a) \text{ for } s > s_0 + a$$

Method. Work out a few basic Laplace transforms (see the table on page 399) and use linearity and shifting to find Laplace transforms of more complicated functions **without integrating anything**.

Examples.

$$L\left(7te^{-t} - 3\right) = 7L\left(te^{-t}\right) - 3L(1) = 7\left(\frac{1}{s+1}\right) - 3\left(\frac{1}{s}\right) \text{ for } s > 0$$

4. Find the Laplace transform of $\sinh(bt) = \dfrac{e^{bt} - e^{-bt}}{2}$