Solutions to the worksheet on inverse Laplace transforms

1a) $L^{-1}\left(\frac{8}{s-2}\right) = 8e^{2t}$

b) $L^{-1}\left(\frac{s}{s^2+4}\right) = \cos 2t$

c) $L^{-1}\left(\frac{8}{s^2+4}\right) = 4L^{-1}\left(\frac{2}{s^2+4}\right) = 4\sin 2t$

this must be the square of the numerator; adjust the numerator to match.

d) $L^{-1}\left(\frac{8}{s-2} - \frac{s}{s^2+4} + \frac{3}{s^4}\right) = 8e^{2t} - \cos 2t + L^{-1}\left(\frac{3}{s^4}\right)$

$= 8e^{2t} - \cos 2t + \frac{t^3}{2}$

$L^{-1}\left(\frac{3}{s^4}\right) = \frac{1}{2}L^{-1}\left(\frac{6}{s^4}\right) = \frac{1}{2}t^3$

$4 = 3+1$ and $3! = 6$ so I want $\frac{6}{s^4}$ because $L^{-1}\left(\frac{6}{s^4}\right) = t^3$.

2 a) $\frac{3-(s+1)(s-2)}{(s+1)(s+2)(s-2)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s-2}$

$*(s+1)$: $\frac{3-(s+1)(s-2)}{(s+2)(s-2)} = A + \frac{B(s+1)}{s+2} + \frac{C(s+1)}{s-2}$

$s=-1$: $A = \frac{3-0(-3)}{1(-3)} = \frac{3}{-3} = -1$

$*(s+2)$ & $s=-2$: $B = \frac{3-(-1)(-4)}{(-1)(-4)} = \frac{-1}{4}$

2a continued.

$$*(s-2) \ \& \ s=2: \quad C = \frac{3-3(0)}{3(4)} = \frac{1}{4}$$

$$L^{-1}\left(\frac{3-(s+1)(s-2)}{(s+1)(s+2)(s-2)}\right) = L^{-1}\left(\frac{-1}{s+1} + \frac{-1/4}{s+2} + \frac{1/4}{s-2}\right)$$

$$= \boxed{-e^{-t} - \frac{1}{4}e^{-2t} + \frac{1}{4}e^{2t}}$$

b) $$\frac{2+(s-2)(3-2s)}{(s-2)(s+2)(s-3)} = \frac{A}{s-2} + \frac{B}{s+2} + \frac{C}{s-3}$$

$$*(s-2) \ \& \ s=2: \quad A = \frac{2+0}{4(1)} = \frac{1}{2}$$

$$*(s+2) \ \& \ s=-2: \quad B = \frac{2+(-4)(7)}{(-4)(-5)} = \frac{-12}{20} = -\frac{3}{5}$$

$$*(s-3) \ \& \ s=3: \quad C = \frac{2+(1)(-3)}{1(5)} = -\frac{1}{5}$$

$$L^{-1}\left(\frac{2+(s-2)(3-2s)}{(s-2)(s+2)(s-3)}\right) = L^{-1}\left(\frac{1/2}{s-2} - \frac{3/5}{s+2} - \frac{1/5}{s-3}\right)$$

$$= \boxed{\frac{1}{2}e^{2t} - \frac{3}{5}e^{-2t} - \frac{1}{5}e^{3t}}$$