Math 260 **Laplace and the Piecewise Continuous Forcing Function** **Due: May 27, 2020**

Video 1. Watch the video on the unit step function and its Laplace transform.

Definition. The **unit step function** is $u(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0 \end{cases}$

Method. The piecewise function $f(t) = \begin{cases} g(t) & \text{if } t < t_1 \\ h(t) & \text{if } t \geq t_1 \end{cases}$ can be expressed as

$$f(t) = g(t) + u(t - t_1)\,[h(t) - g(t)]$$

Theorem. If $t_1 \geq 0$ and $L\left(g(t + t_1)\right)$ exists for $s > s_0$, then $\boxed{L\left(u(t - t_1)g(t)\right) = e^{-st_1} L\left(g(t + t_1)\right)}$ for $s > s_0$.

1. Find the Laplace transforms of the following functions by first re-expressing them using the unit step function the applying the theorem above.

a) $f(t) = \begin{cases} 1 & \text{if } t < 2 \\ t & \text{if } t \geq 2 \end{cases}$

b) $f(t) = \begin{cases} e^{t} & \text{if } t < \ln(2) \\ e^{-t} & \text{if } t \geq \ln(2) \end{cases}$

Video 2. Watch the video on the second shifting theorem.

Theorem (Second shifting theorem). $\boxed{e^{-st_1} L\left(g(t)\right) = L\left(u(t - t_1)g(t - t_1)\right)}$

2. Use the second shifting theorem to fin the inverse Laplace transforms of the following functions.

a) $H(s) = \dfrac{e^{-s}}{s - 2}$. Hint: $\frac{1}{s-2} = L(e^{2t})$.

b) $H(s) = \dfrac{1}{s} + \dfrac{e^{-3s}}{s^2}$

3. Use Laplace transforms to solve the IVP: $y'' - 2y' = \begin{cases} 4, & 0 \le t < 1 \\ 6, & t \ge 1 \end{cases}$, $y(0) = -6$, $y'(0) = 1$.

a) Take the Laplace transform of the entire differential equation (using $L(y'') = s^2 L(y) - sy(0) - y'(0)$ and $L(y') = sL(y) - y(0)$).

b) Sub in the initial conditions and solve for $Y = L(y)$.

c) Take the inverse Laplace transform to get the solution $y(t) = L^{-1}(Y)$.