Video 1. Watch the video on convolutions.

Definition. Let f and g be functions such that if $t < 0$, then $f(t) = 0$ and $g(t) = 0$. The **convolution** of f and g is the function $f * g$ defined by

$$(f * g)(t) = \int_0^t f(\tau)g(t-\tau)d\tau$$

Theorem (Convolution Theorem). $\boxed{L(f * g) = L(f)L(g)}$

1. Use the Convolution Theorem to evaluate the integral $\int_0^2 \tau^7(2-\tau)^5 d\tau$ by:

a) Identifying $h(t) = \int_0^t \tau^7(t-\tau)^5 d\tau$ as the convolution of two functions f and g.

b) Applying the Convolution Theorem to find $L(h)$.

c) Taking the inverse Laplace transform to find $h(t)$.

d) $\int_0^2 \tau^7(2-\tau)^5 d\tau = h(2) =$?

Video 2. Watch the introduction to IVPs involving impulses.

Definition. The **Dirac delta** (or **unit impulse**) function is defined to be the "function" $\delta(t)$ such that

1. $\delta(t) = 0$ if $t \neq 0$
2. $\displaystyle\int_{\infty}^{\infty} \delta(t)dt = 1$

Note that there is no real number y such that $\delta(0) = y$.

Theorem. If $t_0 > 0$, then the solution to the IVP $ay'' + by' + cy = \delta(t - t_0)$, $y(0) = 0$, $y'(0) = 0$, is $\boxed{y = u(t - t_0)w(t - t_0)}$ where $w(t) = L^{-1}\left(\dfrac{1}{as^2 + bs = c}\right)$ (w is often called the **impulse response** of the system).

Method. To solve an IVP of the form $ay'' + by' + cy = f(t) + \alpha\delta(t - t_0)$, $y(0) = k_0$, $y'(0) = k_1$:

1. Find a solution $\hat{y}(t)$ to the IVP $ay'' + by' + cy = f(t)$, $y(0) = k_0$, $y'(0) = k_1$ (use any method);
2. Find the impulse response $w(t)$ of $ay'' + by' + cy = \delta(t - t_0)$, $y(0) = 0$, $y'(0) = 0$;
3. The solution to the original IVP is $y(t) = \hat{y}(t) + \alpha u(t - t_0)w(t - t_0)$.

2. Use the method above to solve the IVP $y'' - 4y = 2e^{-t} + 5\delta(t - 1)$, $y(0) = -1$, $y'(0) = 2$.