Video 1. Watch the introction to linear systems of differential equations.

1. Rewrite the system in matrix form and verify that the given vector function is a solution.

a)

$$\begin{aligned} y_1' &= -4y_1 - 10y_2 \\ y_2' &= 3y_1 + 7y_2 \end{aligned}$$

$$\mathbf{y} = \begin{bmatrix} -5 \\ 3 \end{bmatrix} e^{2t} + \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{t}$$

b)

$$\begin{aligned} y_1' &= -y_1 + 2y_2 + 3y_3 \\ y_2' &= y_2 + 6y_3 \\ y_3' &= -2y_3 \end{aligned}$$

$$\mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} e^{t} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} e^{-t} + \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} e^{-2t}$$

Definition. Matrix multiplication: $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} ax+bz & ay+bw \\ cx+dz & cy+dw \end{bmatrix}$

2. Verify that $Y' = AY$ for $Y = \begin{bmatrix} e^{6t} & e^{-2t} \\ e^{6t} & -e^{-2t} \end{bmatrix}$ and $A = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$.