Math 260 **Solving linear systems** **Due: April 17, 2020**

Video 1. Watch the lesson on solving linear systems of differential equations.

1. Solve the IVP $\mathbf{y}' = \begin{bmatrix} 2 & -4 \\ -1 & -1 \end{bmatrix} \mathbf{y}$, $\mathbf{y}(0) = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$.

2. Find the general solution to $\mathbf{y}' = \begin{bmatrix} 3 & 4 \\ -1 & 7 \end{bmatrix} \mathbf{y}$.

3. Find the general solution to $\mathbf{y}' = \begin{bmatrix} 1 & 2 \\ -4 & 5 \end{bmatrix} \mathbf{y}$.

Video 2. Watch another video on phase planes and describing the solutions.

4. Characterize each system above as a source, a sink, a saddle, a spiral source, a spiral sink, an ellipse, or none of the above.

The following problems are optional, but encouraged

Sometimes a small change in a parameter in an ODE can have a big effect on the behavior of the solutions. This is called a **bifurcation event** (and is a key feature of chaotic systems). We will study the bifurcations that occur in the system

$$\boxed{\mathbf{y}' = \begin{bmatrix} \alpha & 1 \\ -4 & 0 \end{bmatrix} \mathbf{y}}$$

5. Download and run pplane.jar. Set the Display Window so that both x and y range between -2 and 2. Graph the phase plane for $\alpha = -5$, $\alpha = 0$, $\alpha = 2$, $\alpha = 4$, and $\alpha = 8$. (Hint: for $\alpha = 8$ you'll need to enter the system of equations $x' = 8x + y$ and $y' = -4x$). Compare and contrast the phase plane graphs.

6. Find the eigenvalues of $\begin{bmatrix} \alpha & 1 \\ -4 & 0 \end{bmatrix}$ as a function of α (use the quadratic formula).

a) For what values of α does the matrix have distinct real eigenvalues?

b) For what values of α does the matrix have repeated real eigenvalues?

c) For what values of α does the matrix have imaginary eigenvalues?

7. Choose any value of α you want and solve the IVP (note that solutions are hard to find for some choices of α):

$$\mathbf{y}' = \begin{bmatrix} \alpha & 1 \\ -4 & 0 \end{bmatrix} \mathbf{y}, \ \mathbf{y}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

8. Use pplane.jar to plot the solution to your IVP by entering your choice of of α and selecting "Keyboard Input of Initial Value" from the Solution menu and entering $x = 1$ and $y = 0$. Sketch the graph along with dashed lines for the eigenvector(s) of the matrix (if real-valued).

9. Check the "Use current initial values in new graph" box at the bottom left of the equation window of pplane.jar and then graph the phase plane for $\alpha = -5$, $\alpha = 0$, $\alpha = 2$, $\alpha = 4$, and $\alpha = 8$ (this time you should see the graph of the solution to the IVP in addition to the arrows). Compare these graphs with your (and admire the spirals).