LINEAR DIFFERENTIAL EQUATIONS

1 (Completion). We think that the general solution to the differential equation $y' + \left(\frac{3}{x}\right) y = x$ is $y = \dfrac{x^2}{5} + \dfrac{c}{x^3}$.

a) Check that this solution actually works (by differentiating and substituting)
b) Solve the IVP: $y' + \left(\frac{3}{x}\right) y = x$, $y(1) = 2$

Theorem. The **general solution** to the homogeneous linear differential equation $y' + p(x)y = 0$ is

$$\boxed{y = ce^{-P(x)}}$$

where $P(x)$ is an antiderivative of $p(x)$ (any function so that $P'(x) = p(x)$; alternatively, $P(x) = \int p(x)\,dx$). Note that the theorem requires that $p(x)$ be integrable.

Theorem. The general solution to the linear differential equation $y' + p(x)y = f(x)$ is

$$\boxed{y = uy_1}$$

where
a) y_1 is any particular solution to the complementary equation $y' + p(x)y = 0$
b) $u = \displaystyle\int \frac{f(x)}{y_1(x)}\,dx$ (add a constant here).

Method. To solve a linear differential equation:
0. Determine that it is linear by rewriting as $y' + p(x)y = f(x)$
1. Solve the complementary equation $y' + p(x)y = 0$ and choose a solution to be y_1
2. Find $u = \displaystyle\int \frac{f(x)}{y_1(x)}\,dx$
3. Your solution is $y = uy_1$ (check by differentiating and substituting)

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2 (Graded)**.** Solve the IVP: $y' + 2xy = x$, $y(1) = 1$

3 (Completion). Solve the IVP: $(1+x)y' + 2y = \dfrac{\sin x}{1+x}$, $y(0) = 1$