MODELS USING DIFFERENTIAL EQUATIONS

1 (Graded)**.** We saw that the exponential model for population growth $P' = aP$ predicts unbounded population sizes. The logistic model resolves this problem: $P' = aP(1 - bP)$ where a and b are positive constants. If it helps you to solve the problem, you may use $a = 1$ and $b = 0.5$.

a) Draw a phase line and use it to predict the long-term population trends.

b) Use separation of variables to find a solution $P(t)$. (I recommend using partial fractions in the integral; you'll also need to use laws of logarithms to solve for P).

c) Calculate $\lim_{t\to\infty} P(t)$ and compare with your answer for part a.

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2 (Completion)**.** Newton's law of cooling states that the rate of change of an object's temperature is proportional to the difference between its temperature and the temperature of its environment. As a differential equation: $T' = -k(T - T_m)$ where k is a positive constant of proportionality and T_m is the (constant) temperature of the environment.

a) What is the eventual temperature T of the object? Use your understanding of cooling (or heating) to answer (not math).
b) Draw a phase line for the model and verify that it agrees with your answer for part a.
c) Find the general solution to the differential equation.

3 (Completion)**.** A cup of boiling water (212° F) is placed outside. One minute later the temperature of the water is 152° F. After another minute the temperature is 112° F. What is the outside temperature?

4 (Graded). An object with a mass of $m = 10$ kg is launched upward with initial velocity of $v_0 = 60$ m/s. The atmosphere resists the object's motion with a force of 5 Ns/m (that's 5 Newtons for each m/s of speed). The only other force acting on the object is gravity (the acceleration of which is $g = 9.8$ m/s^2 downward). This means that the total force on the object is $F = -mg - 5v$. Newton tells us that $F = ma = mv'$. We now have a differential equation (in the variable v): $\boxed{mv' = -mg - 5v}$

a) Find the terminal velocity of the object (the equilibrium solution to the equation).

b) Find a formula for the velocity of the object.