

EULER'S METHOD

This worksheet concerns the differential equation

$$y' = x + y^2$$

There is no good method to solve this equation by hand.

1. Use the GeoGebra direction field plotter (<https://www.geogebra.org/m/W7dAdgqc>) to plot a direction field. Increase the density of the slope field by pulling the **Density** slider all the way to the right.

- a) Use the **Input** box at the bottom to set $A = (-1, 1)$, then toggle on **Solution A**. Use this curve to estimate $y(0)$ where y is the solution to the IVP $y' = x + y^2$, $y(-1) = 1$.
- b) Use the **Input** box at the bottom to set $B = (-2, 1)$, then toggle on **Solution B**. Use this curve to estimate $y(0)$ where y is the solution to the IVP $y' = x + y^2$, $y(-2) = 1$.

2. Use Euler's method with the following step sizes to repeat the estimates for problem 1.

- a) $h = 0.5$. Do this by hand (share with a friend so you can split up the work).
- b) $h = 0.1$. Use a spreadsheet.
- c) $h = 0.025$. Use a spreadsheet.

A	B	C	D	E
k	x _k	y _k	f(x _k , y _k)	h
0	-1	1	0	0.5
1	-0.5	1	0.5	0.5
2	0	1.25	1.5625	0.5

A	B	C	D	E
k	x _k	y _k	f(x _k , y _k)	h
0	-1	1	0	0.1
1	-0.9	1	0.1	0.1
2	-0.8	1.01	0.2201	0.1
3	-0.7	1.03201	0.365045	0.1
4	-0.6	1.068514	0.541723	0.1
5	-0.5	1.122687	0.760426	0.1
6	-0.4	1.198729	1.036952	0.1
7	-0.3	1.302425	1.39631	0.1
8	-0.2	1.442056	1.879524	0.1
9	-0.1	1.630008	2.556926	0.1
10	-1.4E-16	1.885701	3.555866	0.1

A	B	C	D	E
k	x _k	y _k	f(x _k , y _k)	h
0	-1	1	0	0.025
1	-0.975	1	0.025	0.025
2	-0.95	1.000625	0.05125	0.025
3	-0.925	1.001906	0.078816	0.025
4	-0.9	1.003877	0.107768	0.025
5	-0.875	1.006571	0.138185	0.025
6	-0.85	1.010025	0.170152	0.025
7	-0.825	1.014279	0.203762	0.025
8	-0.8	1.019373	0.239122	0.025
9	-0.775	1.025351	0.276345	0.025
10	-0.75	1.03226	0.315561	0.025
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24	-0.4	1.263379	1.196127	0.025
25	-0.375	1.293282	1.297579	0.025
26	-0.35	1.325722	1.407538	0.025
27	-0.325	1.36091	1.527077	0.025
28	-0.3	1.399087	1.657445	0.025
29	-0.275	1.440523	1.800107	0.025
30	-0.25	1.485526	1.956788	0.025
31	-0.225	1.534446	2.129524	0.025
32	-0.2	1.587684	2.32074	0.025
33	-0.175	1.645702	2.533336	0.025
34	-0.15	1.709036	2.770803	0.025
35	-0.125	1.778306	3.037371	0.025
36	-0.1	1.85424	3.338206	0.025
37	-0.075	1.937695	3.679663	0.025
38	-0.05	2.029687	4.069628	0.025
39	-0.025	2.131428	4.517983	0.025
40	6.04E-16	2.244377	5.037229	0.025

A	B	C	D	E
k	xk	yk	f(xk,yk)	h
0	-2	1	-1	0.5
1	-1.5	0.5	-1.25	0.5
2	-1	-0.125	-0.98438	0.5
3	-0.5	-0.61719	-0.11908	0.5
4	0	-0.67673	0.45796	0.5

A	B	C	D	E
k	xk	yk	f(xk,yk)	h
0	-2	1	-1	0.1
1	-1.9	0.9	-1.09	0.1
2	-1.8	0.791	-1.17432	0.1
3	-1.7	0.673568	-1.24631	0.1
4	-1.6	0.548937	-1.29867	0.1
5	-1.5	0.419071	-1.32438	0.1
6	-1.4	0.286633	-1.31784	0.1
7	-1.3	0.154849	-1.27602	0.1
8	-1.2	0.027246	-1.19926	0.1
9	-1.1	-0.09268	-1.09141	0.1
10	-1	-0.20182	-0.95927	0.1
11	-0.9	-0.29775	-0.81135	0.1
12	-0.8	-0.37888	-0.65645	0.1
13	-0.7	-0.44453	-0.5024	0.1
14	-0.6	-0.49477	-0.35521	0.1
15	-0.5	-0.53029	-0.2188	0.1
16	-0.4	-0.55217	-0.09511	0.1
17	-0.3	-0.56168	0.015482	0.1
18	-0.2	-0.56013	0.113745	0.1
19	-0.1	-0.54876	0.201132	0.1
20	6.38E-16	-0.52864	0.279462	0.1

A	B	C	D	E
k	xk	yk	f(xk,yk)	h
0	-2	1	-1	0.025
1	-1.975	0.975	-1.02438	0.025
2	-1.95	0.949391	-1.04866	0.025
3	-1.925	0.923174	-1.07275	0.025
4	-1.9	0.896355	-1.09655	0.025
5	-1.875	0.868942	-1.11994	0.025
6	-1.85	0.840943	-1.14281	0.025
7	-1.825	0.812373	-1.16505	0.025
8	-1.8	0.783247	-1.18652	0.025
9	-1.775	0.753584	-1.20711	0.025
10	-1.75	0.723406	-1.22668	0.025
A	B	C	D	E
68	-0.3	-0.53992	-0.00848	0.025
69	-0.275	-0.54014	0.016746	0.025
70	-0.25	-0.53972	0.041294	0.025
71	-0.225	-0.53868	0.065181	0.025
72	-0.2	-0.53705	0.088428	0.025
73	-0.175	-0.53484	0.111058	0.025
74	-0.15	-0.53207	0.133096	0.025
75	-0.125	-0.52874	0.154566	0.025
76	-0.1	-0.52488	0.175495	0.025
77	-0.075	-0.52049	0.195908	0.025
78	-0.05	-0.51559	0.215834	0.025
79	-0.025	-0.5102	0.235299	0.025
80	-2.9E-15	-0.50431	0.254331	0.025