UNDETERMINED COEFFICIENTS

Theorem. If y_p is any solution to the differential equation $y'' + p(x)y' + q(x)y = f(x)$ and $\{y_1, y_2\}$ is a fundamental set of solutions to the complementary equation $y'' + p(x)y' + q(x)y = 0$, then a general solution for the original differential equation is

$$\boxed{y = y_p + c_1 y_1 + c_2 y_2}$$

Theorem (Superposition)**.** If y_{p_1} is a particular solution of $y'' + p(x)y' + q(x)y = f_1(x)$ and y_{p_2} is a particular solution of $y'' + p(x)y' + q(x)y = f_2(x)$, then a particular solution of

$$y'' + p(x)y' + q(x)y = f_1(x) + f_2(x)$$

is

$$y_p = y_{p_1} + y_{p_2}$$

Method. To find a solution y_p to the differential equation $ay'' + by' + cy = P(x)$ where $P(x)$ is a polynomial of degree n (and $c \neq 0$): set $y_p = A_n x^n + A_{n-1}x^{n-1} + \cdots + A_0$ and solve for the undetermined coefficients $A_0, A_1, \ldots, A_n$.

Example. Solve the IVP $y'' + 2y' + y = t - 3$, $y(0) = 1$, $y'(0) = -1$.

Method. To find a solution y_p to the differential equation $ay'' + by' + cy = ke^{\alpha x}$: set y_p equal to the first of the following that is not a solution to the complementary equation and solve for the undetermined coefficients.

(1) $y_p = Ae^{\alpha x}$
(2) $y_p = Axe^{\alpha x}$
(3) $y_p = Ax^2e^{\alpha x}$

1 (Completion)**.** Solve the IVP $y'' + 3y' + 2y = e^{-t}$, $y(0) = 1$, $y'(0) = -1$.

Method. To find a solution y_p to the differential equation $ay'' + by' + cy = p\cos(\omega x) + q\sin(\omega x)$: set y_p equal to the first of the following that is not a solution to the complementary equation and solve for the undetermined coefficients.

(1) $y_p = A\cos(\omega x) + B\sin(\omega x)$
(2) $y_p = Ax\cos(\omega x) + Bx\sin(\omega x)$

2 (Graded)**.** Solve the IVP $y'' + y = \frac{1}{3}\cos t$, $y(0) = 0$, $y'(0) = 0$.

3 (Graded)**.** Find a particular solution to $y'' + y = \frac{1}{3}\cos(2t)$

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