THE GAMMA FUNCTION

Definition. The gamma function is defined by $\boxed{\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}\,dx}$

1 (Completion)**.** Investigate the gamma function:

a) Find $\Gamma(1)$
b) Use IBP ($\int udv = uv - \int vdu$) to find $\Gamma(2)$
c) Show that for positive α, $\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$ (use IBP again)

Comment. One consequence of the last fact is that for any natural number n, $\Gamma(n+1) = n!$ (where $n!$ is the factorial: $n! = n(n-1)(n-2)\dots(3)(2)1$). Hence $\Gamma(x)$ can be thought of as a continuous version of the factorial; continuous because the function is defined for every positive real number (and some negative real numbers). For example $\Gamma(1/2) = \sqrt{\pi}$.

2 (Completion)**.** Use the fact that $\Gamma(1/2) = \sqrt{\pi}$ to calculate the following:

a) $\Gamma(3/2)$
b) $\Gamma(-1/2)$

Date: March 29, 2021.