INSTRUCTIONS: Answer all 8 questions on the provided sheets of blank paper, clearly labeling your solutions. Double check to make sure your name is on the front, then staple everything together and turn it in. Show any relevant work for problems whose solution is not a proof; solutions without justification may receive little or no credit. You may refer to the following definitions (as well as any others that you remember).

Definition 1. An integer a is *even* if there is an integer b such that $a = 2b$. Integer a is *odd* if there is an integer b such that $a = 2b + 1$.

Definition 2. Let $a, b \in \mathbb{Z}$. We say that a *divides* b (written $a|b$) if there is an integer c such that $b = ac$.

Definition 3. A natural number n is *prime* if its only positive divisors are 1 and n.

Definition 4. Let $a, b \in \mathbb{Z}$ and $n \in \mathbb{N}$. Then $a \equiv b \pmod{n}$ if $n|(a - b)$.

Definition 5. Let A and B be sets. Then A is a *proper subset* of B (written $A \subsetneq B$) if A is a subset of B and A is not equal to B.

1. Determine if $(P \implies Q) \vee P$ is logically equivalent to Q.

2. Translate the following statement from Math to English and determine if is true or false: $(\forall A \subseteq \mathbb{R})\,(\exists x \in \mathbb{R})\ [x \in A]$.

3. Let $n \in \mathbb{N}$ and let P be the statement "if both n and $n + 1$ are prime, then $n = 2$" (which is true).

a) Determine if the converse of P is true or false.

b) Write the contrapositive of P.

4. Determine if $A \subsetneq B$, $B \subsetneq A$, $B = A$, or no relationship holds between A and B.

a) $A = \{2n - 1 : n \in \mathbb{N}\}$ and $B = \{n \in \mathbb{N} : n \equiv 1 \pmod{2}\}$.

b) $A = \{(x, x^2) : x \in \mathbb{R}\}$ and $B = \mathbb{R}^2$.

5. a) Find two natural numbers a and b such that $a + b \equiv 0 \pmod{7}$.

b) Find an integer c such that $2c \equiv 1 \pmod{7}$.

6. Let $a, b \in \mathbb{Z}$. Prove that if 7 does not divide ab, then 7 divides neither a nor b.

7. Let $a \in \mathbb{Z}$. Prove that if a is odd, then $a + 1$ is even.

8. Let $a, b \in \mathbb{Z}$ and $n \in \mathbb{N}$. Prove that if $a \equiv 0 \pmod{n}$, then $ab \equiv 0 \pmod{n}$.