Definition 1. Integers \(a\) and \(b\) have the same parity if both \(a\) and \(b\) are odd, or both \(a\) and \(b\) are even.

Definition 2. Let \(a\) and \(b\) be integers and \(n\) be a natural number. Then \(a \equiv b \pmod n\) (read “\(a\) is congruent to \(b\) mod \(n\)”) if \(n | (a - b)\).

Instructions: Prove the following. All proofs should either be direct or by contraposition. You may use any propositions from the book, homework, or class that you find helpful; if you do so, please refer to the relevant result clearly (e.g. “as proved in class if \(a^2\) is even, then \(a\) is even”).

1. Let \(a \in \mathbb{Z}\). If \(a^2\) is odd, then \(a\) is odd.

2. Let \(x, y, z \in \mathbb{Z}\). If \(x\) does not divide \(yz\), then \(x\) does not divide \(y\) and \(x\) does not divide \(z\).

3. Let \(x \in \mathbb{R}\). If \(x^2 + 5x < 0\), then \(x < 0\).

4. Let \(a, b \in \mathbb{Z}\). If \(a + b\) is even, then \(a\) and \(b\) have the same parity.

5. Let \(a, b \in \mathbb{Z}\). If \(a + b\) is even and \(ab\) is even, then \(a\) and \(b\) are both even.

6. Let \(x \in \mathbb{Z}\). If \(4\) does not divide \(x^2\), then \(x\) is odd.

7. Let \(x \in \mathbb{Z}\). If \(x\) is odd, then \(8 | (x^2 - 1)\).

8. Let \(a \in \mathbb{Z}\) and \(n \in \mathbb{N}\). If \(a \equiv 1 \pmod n\), then \(a^2 \equiv 1 \pmod n\)

9. For any \(a, b \in \mathbb{Z}\), it follows that \((a + b)^3 \equiv a^3 + b^3 \pmod 3\).

10. Let \(a, b, c \in \mathbb{Z}\) and \(n \in \mathbb{N}\). If \(a \equiv b \pmod n\) and \(a \equiv c \pmod n\), then \(b \equiv c \pmod n\).