Definition 1. Integers a and b have the same *parity* if both a and b are odd, or both a and b are even.

Definition 2. Let a and b be integers and n be a natural number. Then $a \equiv b \pmod{n}$ (read "a is congruent to b mod n") if $n|(a-b)$.

INSTRUCTIONS: Prove the following. All proofs should either be direct or by contraposition. You may use any propositions from the book, homework, or class that you find helpful; if you do so, please refer to the relevant result clearly (e.g. "as proved in class if a^2 is even, then a is even").

1. Let $a \in \mathbb{Z}$. If a^2 is odd, then a is odd.

2. Let $x, y, z \in \mathbb{Z}$. If x does not divide yz, then x does not divide y and x does not divide z.

3. Let $x \in \mathbb{R}$. If $x^2 + 5x < 0$, then $x < 0$.

4. Let $a, b \in \mathbb{Z}$. If $a + b$ is even, then a and b have the same parity.

5. Let $a, b \in \mathbb{Z}$. If $a + b$ is even and ab is even, then a and b are both even.

6. Let $x \in \mathbb{Z}$. If 4 does not divide x^2, then x is odd.

7. Let $x \in \mathbb{Z}$. If x is odd, then $8|(x^2 - 1)$.

8. Let $a \in \mathbb{Z}$ and $n \in \mathbb{N}$. If $a \equiv 1 \pmod{n}$, then $a^2 \equiv 1 \pmod{n}$

9. For any $a, b \in \mathbb{Z}$, it follows that $(a + b)^3 \equiv a^3 + b^3 \pmod{3}$.

10. Let $a, b, c \in \mathbb{Z}$ and $n \in \mathbb{N}$. If $a \equiv b \pmod{n}$ and $a \equiv c \pmod{n}$, then $b \equiv c \pmod{n}$.