NAME(S):
MATH 301 LOGIC SEPTEMBER 15, 2014

1. Determine if the following statements are True or False.

a) Either horses have 4 legs or 17 is not prime.

b) Neither do 3 quarters add up to one dollar, nor do horses have 4 legs.

c) If ducks have webbed feet, then Canada lies south of the equator.

d) If Canada lies south of the equator, then ducks are mammals.

Definition 1. A statement is a *contradiction* if its only possible truth value is "false". A statement is a *tautology* if its only possible truth value is "true".

2. Make a truth table for $P \vee (\neg P)$ and $P \wedge (\neg P)$.

3. Fill in the blank with a statement (written in English) that makes the entire statement a tautology.

a) $[P \wedge (\neg P)] \implies$ ________________________________

b) ________________________________ $\implies [P \vee (\neg P)]$

4. Make a truth table for $P \implies (P \vee Q)$.

5. Make a truth table for $[(P \implies Q) \wedge P] \implies Q$ (this is known as Modus Ponens and is one of the most important rules of deductive logic).

6. Another important tautology is Modus Tollens. Fill in the blank (with something involving P and/or Q) to complete the tautology:

$$[(P \implies Q) \wedge (\neg Q)] \implies \underline{\hspace{3cm}}$$

(there are many ways to fill in the blank to make a tautology, but I'm hoping that you'll pick the simplest completion and get Modus Tollens).

Definition 2. The *converse* of $P \implies Q$ is $Q \implies P$. The *contrapositive* of $P \implies Q$ is $(\neg Q) \implies (\neg P)$.

7. Make up an example of a statement of the form $P \implies Q$ which is true and ...

a) ...has a true converse.

b) ...has a false converse.

c) ...has a true contrapositive.

d) ...has a false contrapositive (if you can't do this, use a truth table to show that $P \implies Q$ is logically equivalent to its contrapositive).

8. Which of the following are logically equivalent?

a) $P \implies Q$

b) $P \vee (\neg Q)$

c) $(\neg P) \vee Q$

9. One of DeMorgan's laws states that $\neg(P \vee Q) = (\neg P) \wedge (\neg Q)$. Use this and your answer to the previous problem to find an alternative expression for $\neg(P \implies Q)$.