Definition 1. A relation R on a set A is an *equivalence relation* if it is reflexive ($\forall a \in A,\ aRa$), symmetric ($\forall a, b \in A,\ aRb \implies bRa$), and transitive ($\forall a, b, c \in A,\ aRb \wedge bRc \implies aRc$). If R is an equivalence relation on A and $a \in A$, then the *equivalence class of* a is

$$[a] = \{x \in A : xRa\}.$$

1. Consider the following relation on $\mathbb{Z}$: $R = \big\{(x, y) : x^2 \equiv y^2 \pmod 7\big\}$.

a) Prove that R is an equivalence relation.

b) Prove that xRy if and only if $x \equiv \pm y \pmod 7$.

c) Find all of the equivalence classes for this relation (note that this is 4 different equivalence classes).

Definition 2. A *partition* of a set A is a set of nonempty subsets of A such that both of the following hold:

1. the union of the subsets is A;
2. the intersection of any two of the subsets is $\emptyset$.

2. Verify that your equivalence classes in 1c form a partition of $\mathbb{Z}$.

3. We proved that for any $n \in \mathbb{N}$, the relation $\equiv \pmod n$ is an equivalence relation on $\mathbb{Z}$. The following questions deal with this relation.

a) What are the equivalence classes of this relation? How many are there?

b) Let $a, b \in \mathbb{Z}$. Prove that for any $x \in [a]$ and any $y \in [b]$, it follows that $[x + y] = [a + b]$.

c) Under the same hypotheses as c, does it follow that $[xy] = [ab]$? Prove you are right.

d) Find the equivalence class of $17(134 - 51)$ for the relation $\equiv \pmod 7$ (this should be easy if you use parts b and c).

e) (A GRE Mathematics Subject Test question). Find the units digit of the standard decimal expansion of 7^{25}.