

1. A well known theorem of geometry states that the sum of the interior angles of a triangle must be  $180^\circ$ .
  - a) Determine the sum of the interior angles of any convex quadrilateral (hint: divide it into two triangles).
  - b) Determine the sum of the interior angles of any convex pentagon (hint: divide it into a triangle and a quadrilateral).
  - c) Let  $n$  be a natural number greater than 2. Make a conjecture about the sum of the interior angles of a convex figure with  $n$  sides. Prove your conjecture.

IF YOU FINISH PROBLEM 1, THEN PROVE AS MANY OF THE FOLLOWING AS YOU CAN. ONE OF YOUR FINAL PORTFOLIO ENTRIES MUST BE A SOLUTION TO ONE OF THE FOLLOWING PROBLEMS.

2. Suppose that each point in the plane  $\mathbb{R}^2$  is colored either red or blue. Prove that there is at least one pair of points that are exactly one unit apart and that have the same color.
3. Prove that for any  $n \in \mathbb{N}$ , a set with  $n$  elements has  $\frac{n(n-1)}{2}$  two-element subsets.
4. Prove that the sum of the cubes of any 3 consecutive natural numbers is divisible by 9.
5. For which natural numbers  $n$  is  $n^2 < 2^n$ ? Prove that you are correct.