

# Statements and open sentences

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$R(f, g)$ :  $f$  is the derivative of  $g$ .

# Famous statements

Goldbach's conjecture (1742): every even integer greater than 2 is the sum of two prime numbers.

Twin primes conjecture: there are infinitely many primes  $p$  such that  $p + 2$  is also prime.

Riemann hypothesis: the nontrivial zeros of the Riemann zeta function have real part equal to  $\frac{1}{2}$ .

Banach-Tarski paradox (1924): A (solid) sphere may be decomposed into finitely many sets which can be rearranged to form two spheres, each of which is just as large as the original sphere. <https://youtu.be/s86-Z-CbaHA>

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5. An integer is even if and only if it is divisible by 2.

# Vocabulary

## Definition

In the statement  $P \implies Q$ ,  $P$  is the **antecedent** and  $Q$  is the **consequent**.

1. If the antecedent is true, then the consequent must also be true.
2. Converse?
3. If the antecedent is false, then the statement is true regardless of the consequent.

# Quantifiers

Open sentence  $P(x)$ .

Statements:

- ▶  $\forall x, P(x)$  "for all  $x$ ,  $P(x)$ "
- ▶  $\exists x, P(x)$  "there is an  $x$  such that  $P(x)$ "

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2. The polynomial  $x^3 + x^2 + x + 1$  has a real root.
3. Every degree three polynomial has a real root.
4.  $\lim_{x \rightarrow a} f(x) = L$  if and only if for every number  $\epsilon > 0$  there is a number  $\delta > 0$  such that

$$|x - a| < \delta \implies |f(x) - L| < \epsilon.$$