#### Quantifiers

#### Definition

 $\lim_{x\to a} f(x) = L$  if (and only if) for every number  $\epsilon > 0$  there is a number  $\delta > 0$  such that

$$0 < |x - a| < \delta \implies |f(x) - L| < \epsilon.$$

What does it mean to say  $\lim_{x\to a} f(x) \neq L$ ?

#### Definition

The  $n^{th}$  Fermat number is

$$F_n = 2^{2^n} + 1$$

(for 
$$n = 0, 1, 2, ...$$
).

► 
$$F_0 = 3$$
  
►  $F_1 = 5$ 

$$= 17$$

►  $F_4 = 65537$ 

► 
$$F_2 = 17$$
  
►  $F_3 = 257$ 

# Conjecture (Fermat 1640)

 $F_n$  is always prime.

### Translations (Eng to Math)

- 1.  $\pi$  is not the root of any polynomial.
- 2. For any number r, if  $x^r = 1$ , then |x| = 1.
- 3. Bertrand's postulate: There's always a prime between n and 2n.

# Translations (Math to Eng)

- 1.  $\forall M \in \mathbb{N}, \ \exists x \in P, \ x > M$  (where P is the set of prime numbers)
- 2.  $\exists X \in \mathcal{P}(\mathbb{N}), \ \forall Y \in \mathcal{P}(\mathbb{N}), \ Y \subseteq X$

3. 
$$\forall x_1, x_2 \in \mathbb{R}, \exists y \in \mathbb{R}, \left[ |x_1 - y| = |x_2 - y| \land (\forall z \in \mathbb{R}, |x_1 - z| = |x_2 - z| \implies z = y) \right]$$