TFAE

Theorem

Let f be a function on $\mathbb{R}$ such that $\lim\limits_{x \to \infty} \left[\dfrac{f(x)}{x} \right]$ exists. The following are equivalent:

1. $\lim\limits_{x \to \infty} \left[\dfrac{f(x)}{x} \right] = 1$

2. $\lim\limits_{x \to \infty} \left[\dfrac{x}{f(x)} \right] = 1$

3. $\lim\limits_{x \to \infty} \left[\dfrac{f(x)}{x} \right] \leq 1$ and $\lim\limits_{x \to \infty} \left[\dfrac{x}{f(x)} \right] \leq 1$

This means $1 \iff 2$ and $1 \iff 3$ and $2 \iff 3$.

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Question

Do we have to do 6 proofs?