NAME(S):
MATH 301 LOGIC SEPTEMBER 10, 2018

Definition. A statement is a **contradiction** if its only possible truth value is False. A statement is a **tautology** if its only possible truth value is True.

1. Make truth tables for the statements $P \vee (\neg P)$ and $P \wedge (\neg P)$.

2. Fill in the blank with a statement (written in English) that makes the entire statement true.

a) $[P \wedge (\neg P)] \implies$ ______________________________

b) ______________________________ $\implies [P \vee (\neg P)]$

3. Make a truth table for the statement $P \implies (P \vee Q)$.

4. Make a truth table for the statement $[(P \implies Q) \wedge P] \implies Q$ (this is known as Modus Ponens).

5. The statement in problem 3 might be written in English as "If P is true, then we know that P or Q must be true." Produce a similar English version of Modus Ponens (the statement in problem 4).

6. Last semester Dr. Axon gave an A to any student who got a perfect score on the final exam. It may be helpful to think of this as a statement of the form $P \implies Q$ where P is "the student got a perfect score on the final" and Q is "the student got an A in the class."

a) Suppose Alex got a perfect score on the final exam in Dr. Axon's class. What can you conclude about Alex's grade in the class? (You're almost certainly using Modus Ponens to make this deduction).

b) Brook didn't get an A in Prof. Axon's class. What can you conclude about Brook's score on the final exam?

c) You probably just used Modus Tollens, another very important tautology. Apply the same reasoning to fill in the blanks below to get Modus Tollens ...

...in English:

If P implies Q, and we know that Q is not true, then ______________________________

...in symbols:

$$[(P \implies Q) \wedge (\neg Q)] \implies \underline{\hspace{3cm}}$$

Definition. The **converse** of $P \implies Q$ is $Q \implies P$. The **contrapositive** of $P \implies Q$ is $(\neg Q) \implies (\neg P)$.

7. a) Write a statement (in English) that has a true converse.

b) Write a statement that has a false converse.

c) Write a statement that has a true contrapositive.

d) Write a statement that has a false contrapositive (or use a truth table to show that $P \implies Q$ is logically equivalent to its contrapositive).