PROOFS I

Definitions. Let $n, m \in \mathbb{Z}$.

a) n is **even** if there is $a \in \mathbb{Z}$ such that $n = 2a$.
b) n is **odd** if there is $a \in \mathbb{Z}$ such that $n = 2a + 1$.
c) m **divides** n (written $m|n$) if there is $a \in \mathbb{Z}$ such that $ma = n$.
d) If $n \geq 2$ and the only positive divisors n are 1 and n, then n is **prime**.
e) If $n \geq 2$ and n is not prime, then n is **composite**.

A. Problems

1. Let $k, m, n \in \mathbb{Z}$. Prove that if $k|m$ and $k|(m + n)$, then $k|n$.

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2. Let $m, n \in \mathbb{Z}$.

a) Prove that if either m or n is even, then mn is even.

b) State the converse of "if either m or n is even, then mn is even". Note that the converse has the same implicit quantifiers as the original statement.

c) Is the converse true or false? (No need to prove it).

d) It is also true that if $6|m$ or $6|n$, then $6|(mn)$. Is the converse of this statement true? Explain your reasoning.

Challenge (write your solution on a separate sheet of paper)**.** Prove that if $n \in \mathbb{N}$ and $n \geq 2$, then the numbers $n! + 2$, $n! + 3$, $n! + 4$, $\ldots$, $n! + n$ are all composite. (This means that $n! + 2$, $n! + 3$, $n! + 4$, $\ldots$, $n! + n$ is a sequence of $n - 1$ consecutive composite numbers, thus showing that there are arbitrarily large gaps between prime numbers).