PROOFS III

Theorem 1. *Let $a, b \in \mathbb{Z}$. If 7 does not divide ab, then 7 divides neither a nor b.*

1. Write the **contrapositive** and the **negation** of Theorem 1.

Two proofs of Theorem 1 are given below. Both proofs basically work, but neither is perfect. Read the proofs carefully and note places where they could be improved. Then write a perfect proof of the theorem (using whatever approach you deem best).

> *Proof.* Suppose $a, b \in \mathbb{Z}$ and $7|a$ or $7|b$. By definition, there is an n such that $7n = a$ or $7n = b$. Hence $7na$ or $7nb$ equals ab. By definition $7|(ab)$. This proves the contrapositive. □

> *Proof by contradiction.* Suppose $7 \nmid (ab)$ and $7|a$ or $7|b$. Without loss of generality, suppose $7|a$. Then $ab = 7xb$ for some $x \in \mathbb{Z}$. By definition $7|(ab)$, which is a contradiction. □

2. Write a perfect proof of the theorem.

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3. Prove that for every $n \in \mathbb{N}$, $4 \nmid (n^2 + 2)$.