PROOF AND DISPROOF

1. Prove or disprove the statements:

a) $\boxed{\forall x, y \in \mathbb{R},\ (x+y)^2 = x^2 + y^2}$

Solution. The statement is false. A counterexample: $x = 1$ and $y = 1$.

b) $\boxed{\forall x, y \in \mathbb{R},\ (x+y)^2 \neq x^2 + y^2}$

Solution. The statement is false. A counterexample: $x = 0$ and $y = 0$.

2. Let $a, b \in \mathbb{Z}$. Prove or disprove the statement: $\boxed{\text{If } a|b^2, \text{ then } a|b.}$

Solution. The statement is false. A counterexample: $a = 4$ and $b = 2$.

3. Let A, B, and C be sets. Prove or disprove the statement: $\boxed{\text{If } C \subseteq B, \text{ then } (A - B) \subseteq (A - C).}$

Solution. The statement is true. Proof: Let A, B, and C be sets such that $C \subseteq B$. Let $x \in A - B$. By definition $x \in A$ and $x \notin B$. Because $C \subseteq B$, it follows that $x \notin C$. We now know that $x \in A$ and $x \notin C$. Hence $x \in A - C$. Therefore $(A - B) \subseteq (A - C)$.

Date: October 19, 2018.